EE 641 Midterm Exam

October 12, Fall 2018

Name: ______________________________

Instructions

The following is an in-class closed-book exam.

- This exam contains 3 problems worth a total of 100 points.
- You may not use any notes, textbooks, or calculators.
- You are allowed up to 55 minutes to complete the exam.

Good luck.

Problem 1. (35pt)

Let X and W be independent Gaussian random vectors of dimension p such that $X \sim N(0, R_x)$ and $W \sim N(0, R_w)$, and let θ be a deterministic vector of dimension p.

a) First assume that $Y = \theta + W$, and calculate the ML estimate of θ given Y .

For the next parts, assume that $Y = X + W$.

b) Calculate an expression for $p_{x|y}(x|y)$, the conditional density of X given Y.

c) Calculate the MMSE estimate of X when $Y = X + W$.

d) Calculate an expression for the conditional variance of X given Y.

Problem 2. (35pt)

Let $X \in \Re^N$ be a Gaussian random vector with $X \sim N(0, B^{-1})$, and let

$$Y = AX + W$$

where $W \sim N(0, \sigma^2 I)$. Find expressions for the following.

a) $\mathbb{E}[XX^t]$

b) $\mathbb{E}[X_i | X_j \text{ for } j \neq i]$

c) The MAP of X given Y.

d) The MMSE of X given Y.

Problem 3. (30pt)
Let $B \in \Re^{n \times n}$ be the precision matrix (i.e., the inverse covariance) of a zero-mean GMRF, and let $\mathcal{P}$ be the set of all unique unordered pairs, i.e., $\mathcal{P} = \{\{i, j\} : 1 \leq i, j \leq n\}$.
Then consider the following equality

$$x^t B x = \sum_{i=1}^{n} a_i x_i^2 + \sum_{\{i,j\} \in \mathcal{P}} b_{i,j} |x_i - x_j|^2$$

Find expressions for the values of the coefficients a_i and $b_{i,j}$ and prove that the equality holds for all $x \in \Re^n$.