EE 641 Midterm Exam

October 24, Fall 2016

Name: _____

Instructions

The following is an in-class closed-book exam.

- This exam contains 3 problems worth a total of 100 points.

- You may not use any notes, textbooks, or calculators.

- You are allowed up to 55 minutes to complete the exam.

Good luck.

Problem 1. (30pt)

Let $\{X_i\}_{i=1}^n$ be i.i.d. random variables with distribution

$$P\{X_i = k\} \;\; = \;\; \pi_k$$

where $\sum_{k=1}^m \pi_k = 1$. Compute the ML estimate of the parameter vector $\theta = [\pi_1, \cdots, \pi_m]$. (Hint: You may use the method of Lagrange multipliers to calculate the solution to the constrained optimization.)

Problem 2. (35pt)

Let X_s be a zero-mean GMRF on a finite general lattice $s \in S$. Let X be a vector of dimension $N = |S|$ containing all the elements of X_s in some fixed order, and denote the inverse covariance of X as

$$B = \left(\mathbb{E}[XX^t] \right)^{-1} .$$

a) Write an expression for $p(x)$, the PDF of X in terms of B.

b) If ∂s denotes the neighborhood system of the MRF, then show that if $r \notin \partial s$ and $r \neq s$, then $B_{r,s} = B_{s,r} = 0$.

c) Show that we can define a valid (but possibly different) neighborhood system for this GMRF as

$$\partial s = \{ r \in S : B_{r,s} \neq 0 \text{ and } r \neq s \} .$$

Problem 3. (35pt)

Consider the function

$$f(x) = |x - x_r|^{1.1} \ ,$$

for $x \in \mathbb{R}$.

a) Sketch a plot of $f(x)$ when $x_r = 1$.

b) Sketch a good surrogate function, $f(x; x')$, for $x_r = 1$ and $x' = 2$.

c) Determine a general expression for the surrogate function $f(x; x')$ that works for any value of x_r and x'.

d) Assuming the objective is to minimize the expression

$$f(x) = \sum_{r \in \partial s} |x - x_r|^{1.1} \ ,$$

for $x \in \mathbb{R}$, specify an iterative algorithm in terms of the surrogate function $f(x; x')$ that will converge to the global minimum of the function.