EE 641 Midterm Exam

October 23, Fall 2014

Name: ______________________________

Instructions

The following is an in-class closed-book exam.

- This exam contains 3 problems worth a total of 110 points.
- You may not use any notes, textbooks, or calculators.
- You are allowed up to 75 minutes to complete the exam.

Good luck.

Problem 1. (30pt)

Let $X \sim N(0, R)$ where R is a $p \times p$ symmetric positive-definite matrix. Further define the precision matrix, $B = R^{-1}$, and use the notation

$$B = \begin{bmatrix} 1/\sigma^2 & A \\ A^t & C \end{bmatrix},$$

where $A \in \mathbb{R}^{1\times(p-1)}$ and $C \in \mathbb{R}^{(p-1)\times(p-1)}$.

a) Calculate the marginal density of X_1, the first component of X.

b) Calculate the conditional density of X_1 given all the remaining components, $Y = [X_2, \cdots, X_p]^t$.

c) What is the conditional mean and covariance of X_1 given Y?

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Problem 2. (40pt)

Let X_n be a 1-D zero-mean stationary Gaussian AR process with MMSE causal prediction filter given by $h_n = \rho\delta_{n-1}$ and causal prediction variance σ_c^2.

a) Calculate, $S_X(\omega)$, the power spectral density of the random process.

b) Calculate, $R_X(n)$, the time autocorrelation of the random process.

c) Sketch plots $S_X(\omega)$ and $R_X(n)$ for $\rho = 0.95$.

d) Calculate (σ_{NC}^2, g_n) the noncausal prediction variance and the noncausal prediction filter for the equivalent GMRF.

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Problem 3. (40pt)

Consider the optimization problem

$$\hat{x} = \arg \min_{x \in \mathbb{R}^N} \left\{ ||y - Ax||_\Lambda^2 + x^t B x \right\}$$

where A is a nonsingular $N \times N$ matrix, B is a positive-definite $N \times N$ matrix, and Λ is a diagonal and positive-definite matrix..

a) Derive a closed form expression for the solution.

b) Calculate an expression for the gradient descent update using step size $\mu \geq 0$.

c) Calculate an expression for the update of gradient descent with line search.

d) Calculate an expression for the coordinate descent update.

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