EE 641 Midterm Exam

October 18, Fall 2013

Name: ______________________________

Instructions

The following is an in-class closed-book exam.

- This exam contains 3 problems worth a total of 108 points.
- You may not use any notes, textbooks, or calculators.
- You are allowed 50 minutes to complete the exam. Hand in the exam after that period **whether or not you have completed it.**

Good luck.

Problem 1. (36pt)

Let $X \sim N(0, R)$ where R is a $p \times p$ symmetric positive-definite matrix with an eigen decomposition of the form $R = E\Lambda E^t$.

a) Calculate the covariance of $\tilde{X} = E^t X$, and show that the components of $\tilde{X}$ are jointly independent Gaussian random variables.

b) Show that if $Y = E\Lambda^{1/2}W$ where $W \sim N(0, I)$, then $Y \sim N(0, R)$.

c) How can this result be of practical value?

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Problem 2. (36pt)

Let X_s be a zero-mean 2-D Gaussian AR process indexed by $s = (s_1, s_2)$, and let the MMSE causal prediction filter be given by

$$h_s = \rho \delta_{s_1 - 1, s_2} + \rho \delta_{s_1, s_2 - 1}$$

and the causal prediction variance be given by σ_C^2. Determine (σ_{NC}^2, g_s) the noncausal prediction variance and the noncausal prediction filter.

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Problem 3. (36pt)

Show that if X is a zero-mean GMRF with inverse covariance, B, then $B_{s,r} = 0$ if and only if x_s is conditionally independent of X_r given $\{X_i : \text{ for } i \neq s, r\}$.

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