EE 641 Final Exam

Fall 2013

Name: ______________________________

Instructions

- This exam contains 4 problems worth a total of 100 points.
- You may not use any notes, textbooks, or calculators.
- Answer questions precisely and completely. Credit will be subtracted for vague answers.

Good luck.

Problem 1. (25pt)
Show that if X is distributed as

$$p(x) = \frac{1}{z} \exp \left\{ - \sum_{\{s,r\} \in \mathcal{P}} b_{s,r} \rho(x_s - x_r) \right\} ,$$

where $\rho(\cdot)$ is a potential function and $\mathcal{P}$ is a set of pairwise cliques, then the conditional distribution of X_s given X_r for $r \neq s$ is given by

$$p(x_s | x_r \ r \neq s) = \frac{1}{z} \exp \left\{ - \sum_{r \in \partial s} b_{s,r} \rho(x_s - x_r) \right\} ,$$

and give an expression for z.

Problem 2.(25pt)

Let $x \in \Re^N$ and $A \in \Re^{N \times N}$ be a full rank matrix.

a) Calculate a closed form expression for the ICD update when the MAP cost function has the form.

$$f(x; y) = ||y - Ax||^2 + ||x||_1$$

where $||x||_1$ is the L_1 norm of x, and A has rank N.

b) Prove that $f(x; y)$ is a strictly convex function of x.

c) Prove that this function has a unique global minimum which is also a local minimum.

Problem 3.(25pt)

Let X_n be N i.i.d. random variables with $P\{X_n = i\} = \pi_i$ for $i = 0, \cdots, M-1$. Also, assume that Y_n are conditionally independent given X_n and that the conditional distribution of Y_n given X_n is distributed as $N(\mu_{x_n}, \gamma_{x_n})$. Derive an EM algorithm for estimating the parameters $\{\pi_i, \mu_i, \gamma_i\}_{i=0}^{M-1}$.

Problem 4.(25pt)

Consider a birth-death process for which the state of the Markov chain, X_n, takes on values in the set $\{0, \cdots, M-1\}$, so that $P_{i,i+1} = \lambda$ is the birth rate for $i = 0, \cdots, M-2$; and $P_{i,i-1} = \mu$ is the death rate for $i = 1, \cdots, M-1$.

a) Calculate, $P_{i,j}$, the transition probabilities of the homogeneous Markov chain.

b) Calculate, π_i, the stationary distribution of the Markov chain.