EE 641 Midterm Exam

November 19, Fall 2008

Name: _____

Instructions

The following is an in-class closed-book exam.

- This exam contains 3 problems worth a total of 108 points.
- You may not use any notes, textbooks, or calculators.

Good luck.

Problem 1. (36pt)
The EM algorithm is based on the relationship that

$$\log p(y|\theta) = Q(\theta, \theta') + H(\theta, \theta') \ .$$

a) Give the definition for the Q function.
b) Specify an important mathematical property of the H function. (A proof is not required.)
c) State the basic EM update equation in terms of the Q function.
d) Use the important property of the H function (from part b) to show that each update of the EM algorithm increases the value of $\log p(y|\theta)$.

Problem 2. (36pt)
Let $\{X_n\}_{n=1}^N$ be a i.i.d. random varibles with distribution

$$P\{X_n = m\} = \pi_m \ ,$$

where $\sum_{m=0}^{M-1} \pi_m = 1$. Also, let Y_n be conditionally i.i.d. random vectors given X_n, with multivariate Gaussian conditional distribution $N(\mu_m, R_m)$ when $X_n = m$.
a) Write out the density function for the vector Y.
b) What are the natural sufficient statistics for the complete data (X, Y)?
c) Give an expression for the ML estimate of the parameter $\theta = (\pi_0, \mu_0, R_0, \cdots, \pi_{M-1}, \mu_{M-1}, R_{M-1})$ given the complete data (X, Y).
d) Give the EM update equations for computing the ML estimate of the parameter $\theta = (\pi_0, \mu_0, R_0, \cdots, \pi_{M-1}, \mu_{M-1}, R_{M-1})$ given the incomplete data Y.

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Problem 3. (36pt)
Let $\{X_n\}_{n=1}^N$ be a i.i.d. random varibles with distribution

$$P\{X_n = m\} = \pi_m \ ,$$

where $\sum_{m=0}^{M-1} \pi_m = 1$. Also, let Y_n be conditionally i.i.d. random variables given X_n, with Poisson conditional distribution

$$p(y_n|x_n = m) = \frac{\lambda_m^{y_n} e^{-\lambda_m}}{y_n!} \ .$$

a) Write out the density function for the vector Y.
b) What are the natural sufficient statistics for the complete data (X, Y)?
c) Give an expression for the ML estimate of the parameter $\theta = (\pi_0, \lambda_0, \cdots, \pi_{M-1}, \lambda_{M-1})$ given the complete data (X, Y).
d) Give the EM update equations for computing the ML estimate of the parameter $\theta = (\pi_0, \lambda_0, \cdots, \pi_{M-1}, \lambda_{M-1})$ given the incomplete data Y.

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