EE 641 Final Exam

Fall 2008

Name: ______________________ Starting time: ______________________

Ending time: ______________________

Instructions

The following are important rules for this take home exam.

- Accurately fill in a start time and ending time for your exam.
- You are allowed 24 hours to complete the exam. Hand in the exam after that period **whether or not you have completed it.**
- You should not discuss these problems with any other person. In addition, you should not communicate with any other student in the class during the test period.
- Your are allowed to use all class notes and handouts (including notes from homeworks and labs), any material posted on the official course web page, and the course text book from EE600 or an equivalent graduate course.
- Your are not allowed to use any other supplementary information, including sources from the library, publications not handed out in class, or google searches from the web.
- You must hand in your completed exam by Monday December 22 at 9:00AM.
- If you can not physically hand in your exam, you may email me a scanned copy of the exam. Please make sure the scan is readable, and make sure you get absolute confirmation of my receipt of the exam.
- If you have any questions, call me at home 463-4378 (8:00AM to 8:00PM); or office 494-0340, or send email at bouman@purdue.edu.

Good luck.

Problem 1.(33pt)

a) Let X_n for $n = 0$ to $N-1$ be i.i.d. p dimensional Gaussian random vectors with unknown mean μ and convariance R. Derive the maximum likelihood estimate for μ and R. (Hint: The outline of the proof is in your class notes. It was a homework problem, and if that isn't good enough, it should be in the 600 text.)

b) Let $\{X_n\}_{n=0}^{N}$ be a Markov chain with initial probability $P\{X_0 = m\} = \rho_m$, and transition probability

$$P\{X_n = j | X_{n-1} = i\} = P_{i,j} \ .$$

Derive the ML estimate of the parameters ρ_m and $P_{i,j}$.

Problem 2.(33pt) Consider a p dimensional random vector Y with density

$$p(y|B) = (2\pi)^{-p/2}|B|^{1/2} \exp\left\{-\frac{1}{2}y^t By\right\}$$

a) What is the mean of Y?

b) What is the conditional expectation, $E\left[Y_n|Y_k \text{ for } k \neq n\right]$?

b) What is the conditional variance, $Var\left[Y_n|Y_k \text{ for } k \neq n\right]$?

Problem 3.(34pt) Consider a distribution of the form

$$p(x) = \frac{1}{z} \exp\left\{ - \sum_{\{i,j\} \in \mathcal{P}} \rho(x_i - x_j) \right\} ,$$

where $\rho(\Delta)$ is a positive, continuous, and symmetric function of Δ, and $\mathcal{P}$ is the set of pairwise neighbors.

a) What choice of $\rho(\Delta)$ makes X a Gaussian random field?

b) Give an example of a **non-convex** function $\rho(\Delta)$ that might be useful when $p(x)$ is used as a prior model. Explain the advantages of your choice, and when it is best used.

c) Give an example of a **convex** function $\rho(\Delta)$ that might be useful when $p(x)$ is used as a prior model. Explain the advantages of your choice, and when it is best used.