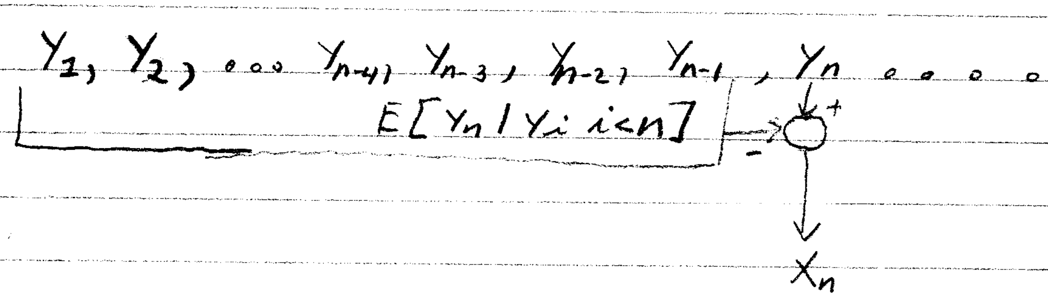
Lecture 3

Whitening

Let Y be a zero mean Gaussian discrete time random Process.



- Use all previous values of Y_i $i<n$ to predict Y_n

$$\hat{Y}_n = E[Y_n / Y_{n-1}, Y_{n-2}, \ldots, Y_1]$$

- Define the prediction errors X_n

$$X_n = Y_n - \hat{Y}_n$$

$$= Y_n - \underbrace{E[Y_n | Y_i \; i<n]}_{\text{predictor}}$$

prop 1) $E[Y_n | Y_i \; i<n] = \sum_{i=1}^{n-1} a_{ni} Y_i$

Gaussian $\Rightarrow$ Linear for some a_{ni}
$\Rightarrow$ X_i's Gaussian

property 2) $X_n \underset{\text{independent}}{\perp\!\!\!\perp} Y_i$ for $i < n$

For Gaussian, enough to be uncorrelated

$$(i<n) \quad E[X_n Y_i] = E[(Y_n - E[Y_n \mid Y_j\ j<n])Y_i]$$

$$= E[\,Y_n Y_i - E[Y_n Y_i \mid \underbrace{Y_j\ j<n}_{\text{includes } Y_i}]\,]$$

$$= E[Y_n Y_i] - E[E[Y_n Y_i \mid Y_j\ j<n]]$$

$$= 0$$

prop 3) $X_n \perp\!\!\!\perp (Y_{n-1}, Y_{n-2}, \dots, Y_1)$

Because its pairwise independent and Gaussian prop 4) $X_n \perp\!\!\!\perp X_j$ for $n \neq j$

assume that $j < n$ ($n < j$ the same)

$$E[X_n X_j] = E\left[X_n\left(Y_j - \sum_{i=1}^{j-1} a_{ji} Y_i\right)\right]$$

$$= \underset{\overset{\shortparallel}{0}}{E[X_n Y_j]} - \sum_{i=1}^{j-1} a_{ji} \underset{\overset{\shortparallel}{0}}{E[X_n Y_i]}$$

$$= 0$$

prop 5) Since the X_i's are independent, we can write the distribution

Let $\sigma_i^2 = E[X_i^2]$

$$p_x(x) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\,\sigma_i} \exp\left\{-\frac{1}{2\sigma_i^2} X_i^2\right\}$$