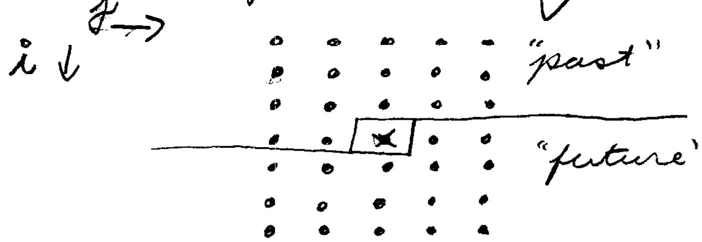
$\boxed{\text{Lecture 6}}$

2D AR Process

Ordering/causality?

Y_{ij}



Raster Ordering — $\begin{bmatrix} Y_{11} \\ \vdots \\ Y_{1N} \\ Y_{21} \\ \vdots \\ Y_{2N} \\ Y_{N1} \\ \vdots \\ Y_{NN} \end{bmatrix}$

past - Nonsymetric half plain (NHP)

$$Y_{ij} = X_{ij} + \sum_{k=1}^{p} \sum_{l=-p}^{p} h_{k,l}\, Y_{i-k,j-l} + \sum_{l=1}^{p} h_{0,l}\, Y_{i,j-l}$$

Let $s = (s_1, s_2)$
$r = (r_1, r_2)$

$$s < r \Leftrightarrow (s_1 < r_1) \text{ or } (s_1 = r_1 \text{ and } s_2 < r_2)$$

$$Y_s = X_s + \sum_{r>0} h_r\, Y_{s-r}$$

($d_r = 0$ outside prediction windo

$$\sigma_Y(\bar{\omega}) = \left| \frac{\sigma^2}{1 - H(\bar{\omega})} \right|^2 \qquad \bar{\omega} = (\omega_1, \omega_2)$$

$$H(\bar{\omega}) = \sum_{s} h_s e^{-j\bar{\omega}\cdot s} \qquad \bar{\omega}\cdot s = \omega_1 s_1 + \omega_2 s_2$$

2D DTFT

$$= \sum_{s_1 s_2} h_{(s_1, s_2)} e^{j(\omega_1 s_1 + \omega_2 s_2)}$$

Matrix Representation of 2-D Filter

$$\overrightarrow{\begin{matrix} h_{11} & h_{10} & h_{1-1} \\ h_{01} & h_{00} & \end{matrix}} \qquad \text{order } P = 1$$

$$X_s = \sum_{r \geq (0,0)} h_r \, Y_{s-r}$$

$\bar{Y}_i$ – i^{th} row of Y

$$Y = \begin{bmatrix} \bar{Y}_1 \\ \vdots \\ \bar{Y}_N \end{bmatrix} \qquad X = \begin{bmatrix} \bar{X}_1 \\ \\ \bar{X}_N \end{bmatrix}$$

$$X = AY \qquad A = ?$$

$$\begin{aligned} \bar{X}_1 &= A_0 \bar{Y}_1 \\ \bar{X}_2 &= A_0 \bar{Y}_2 + A_1 \bar{Y}_1 \\ \bar{X}_3 &= A_0 \bar{Y}_3 + A_1 \bar{Y}_2 \\ &\vdots \\ \bar{X}_N &= A_0 \bar{Y}_N + A_1 \bar{Y}_{N-1} \end{aligned}$$

$$
\begin{bmatrix} \bar{X}_1 \\ \vdots \\ \bar{X}_N \end{bmatrix} = \begin{bmatrix} A_0 & & & 0 \\ A_1 & \ddots & & \\ & \ddots & \ddots & \\ 0 & & A_1 & A_0 \end{bmatrix} \begin{bmatrix} \bar{Y}_1 \\ \vdots \\ \bar{Y}_N \end{bmatrix}
$$

Vertical Causal $\Rightarrow$ Block lower triangular
Vert. invariant $\Rightarrow$ Block Toeplitz
Vert. FIR $\Rightarrow$ Block $P+1$ diagonal

$$
A_0 = \begin{bmatrix} h_{00} & & & 0 \\ h_{01} & \ddots & & \\ & \ddots & \ddots & \\ 0 & & h_{01} & h_{00} \end{bmatrix}
$$

$$
A_1 = \begin{bmatrix} h_{10} & h_{1-1} & & & 0 \\ h_{11} & h_{10} & h_{1-1} & & \\ & \ddots & \ddots & \ddots & h_{1-1} \\ & 0 & & h_{11} & h_{10} \end{bmatrix}
$$

Horizontal Causal $\Rightarrow$ A_0 lower triangular
Hor. Invariant $\Rightarrow$ A_i Toeplitz
Hor. FIR $\Rightarrow$ A_0 $P+1$ diagonal
A_i $2P+1$ diag.

Computation of Parameters for 2-D AR Model

Let $S = \{(i, j) : i = \text{row},\ j = \text{column of image}\}$

y_s = pixel at position $s = (s_1, s_2)$

$$
\begin{array}{ccccc}
y(s_1-p, s_2-p) & \cdots & y(s_1, s_2-p) & \cdots & y(s_1+p, s_2-p) \\
\vdots & & \vdots & & \vdots \\
 & & y(s_1, s_2-1) & & y(s_1+p, s_2-1) \\
y(s_1-p, s_2) & \cdots\ y(s_1-1, s_2) & y(s_1, s_2) & \cdots & y(s_1+p, s_2)
\end{array}
$$

Define

$$
Z_s = \begin{bmatrix} y(s_1-1, s_2) \\ \vdots \\ y(s_1-p, s_2) \\ y(s_1+p, s_2-1) \\ \vdots \\ y(s_1-p, s_2-1) \\ \vdots \\ \vdots \\ y(s_1+p, s_2-p) \\ \vdots \\ y(s_1-p, s_2-p) \end{bmatrix}
$$

h = column vector of corresponding filter coefficients

Then

$$X_s = Y_s - h^t Z_s$$