Connected Component Analysis

- Once region boundaries have been detected, it is often useful to extract regions which are not separated by a boundary.
- Any set of pixels which is not separated by a boundary is call connected.
- Each maximal region of connected pixels is called a connected component.
- The set of connected components partition an image into segments.
- Image segmentation is an useful operation in many image processing applications.

Connected Neighbors

- Let ∂s be a neighborhood system.
  - 4-point neighborhood system
  - 8-point neighborhood system
- Let $c(s)$ be the set of neighbors that are connected to the point s.

  For all s and r, the set $c(s)$ must have the properties that
  - $c(s) \subset \partial s$
  - $r \in c(s) \Leftrightarrow s \in c(r)$
- Example:
$$c(s) = \{r \in \partial s : X_r = X_s\}$$
- Example:
$$c(s) = \{r \in \partial s : |X_r - X_s| < Threshold\}$$
- In general, computation of $c(s)$ might be very difficult, but we won't worry about that now.

Connected Sets

- Definition: A region $R \subset S$ is said to be connected under $c(s)$ if for all $s, r \in R$ there exists a sequence of M pixels, $s_1, \cdots, s_M$ such that

$$s_1 \in c(s), s_2 \in c(s_1), \cdots, s_M \in c(s_{M-1}), r \in c(s_M)$$

i.e. there is a connected path from s to r.

Example of Connect Sets

- Consider the following image X_s

$$
\begin{array}{cccccc}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{array}
$$

$$
\begin{aligned}
S_1 &= \{s : X_s = 1\} \\
S_0 &= \{s : X_s = 0\}
\end{aligned}
$$

- Define $c(s) = \{r \in \partial s : X_r = X_s\}$
- Result
  - 4-point neighborhood $\Rightarrow$ S_0 and S_1 are not connected sets
  - 8-point neighborhood $\Rightarrow$ S_0 and S_1 **are** connected sets!

Region Growing

- Idea - Find a connected set by growing a region from a seed point s_0
- Assume that $c(s)$ is given

$ClassLabel = 1$
Initialize $Y_r = 0$ for all $r \in S$
ConnectedSet($s_0, Y, ClassLabel$) {
  $B \leftarrow \{s_0\}$
  While B is not empty {
    $s \leftarrow$ any element of B
    $B \leftarrow B - \{s\}$
    $Y_s \leftarrow ClassLabel$
    $B \leftarrow B \bigcup \{r : r \in c(s)$ and $Y_r = 0\}$
  }
  return(Y)
}

Region Growing Example (1)

The list of
$(i, j) \in B$
(0,0)

The image X

		j 0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j 0	1	2	3	4
i	0	1	0	0	0	0
	1	0	0	0	0	0
	2	0	0	0	0	0
	3	0	0	0	0	0
	4	0	0	0	0	0

Region Growing Example (2)

The list of
$(i, j) \in B$
(1,0)

The image X

	j 0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

	j 0	1	2	3	4
i 0	1	0	0	0	0
1	1	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0

Region Growing Example (3)

The list of $(i, j) \in B$

(1,1)

The image X

	j 0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

	j 0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0

Region Growing Example (4)

The list of
$(i,j) \in B$
(2,1)

The image X

		j=0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j=0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	0	0	0
	3	0	0	0	0	0
	4	0	0	0	0	0

Region Growing Example (5)

The list of $(i,j) \in B$

(3,1)

(2,2)

The image X

i \ j	0	1	2	3	4
0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

i \ j	0	1	2	3	4
0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	0	0	0
4	0	0	0	0	0

Region Growing Example (6)

The list of $(i, j) \in B$

(4,1)
(3,2)
(2,2)

The image X

	j=0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

	j=0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	0

Region Growing Example (7)

The list of $(i,j) \in B$

(3,2)

(2,2)

The image X

i \ j	0	1	2	3	4
0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

i \ j	0	1	2	3	4
0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	0

Region Growing Example (8)

The list of
$(i, j) \in B$
(2,2)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Region Growing Example (9)

The list of
$(i, j) \in B$
empty

The image X

		j 0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j 0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Connected Components Extraction

- Iterate through each pixel in the image.
- Extract connected set for each unlabeled pixel.

$ClassLabel = 1$
Initialize $Y_r = 0$ for $r \in S$
For each $s \in S$ {
    if($Y_s = 0$) {
        ConnectedSet($s, Y, ClassLabel$)
        $ClassLabel \leftarrow ClassLabel + 1$
    }
}

Connected Components Extraction Example (1)

$s = (i, j);$
$ClassLabel$
$(0, 0); 1$

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Connected Components Extraction Example (2)

$s = (i, j);$
$ClassLabel$
$(0, 1); 2$

The image X

	j=0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

	j=0	1	2	3	4
i 0	1	2	2	2	2
1	1	1	2	2	2
2	0	1	1	2	2
3	0	1	1	2	2
4	0	1	2	2	0

Connected Components Extraction Example (3)

$s = (i, j);$
$ClassLabel$
$(2, 0); 3$

The image X

	j 0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

	j 0	1	2	3	4
i 0	1	2	2	2	2
1	1	1	2	2	2
2	3	1	1	2	2
3	3	1	1	2	2
4	3	1	2	2	0

Connected Components Extraction Example (4)

$s = (i, j);$
$ClassLabel$
$(4, 4); 4$

The image X

		j 0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j 0	1	2	3	4
i	0	1	2	2	2	2
	1	1	1	2	2	2
	2	3	1	1	2	2
	3	3	1	1	2	2
	4	3	1	2	2	4