### **Continuous Time Fourier Transform (CTFT)**

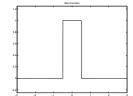
$$F(f) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}dt$$

$$f(t) = \int_{-\infty}^{\infty} F(f)e^{j2\pi ft}df$$

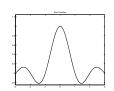
- f(t) is continuous time. (Also known as continuous parameter.)
- F(f) is a continuous function of frequency  $-\infty < f < \infty$ .

## **Useful Continuous Time Signal Definitions**

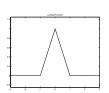
• Rect function:  $rect(t) = \begin{cases} 1 & \text{for } |t| \le 1/2 \\ 0 & \text{otherwise} \end{cases}$ 



- Step function:  $\mathbf{u}(t) = \begin{cases} 1 & \text{for } t \ge 0 \\ 0 & \text{for } t < 0 \end{cases}$
- Sign function:  $\operatorname{sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t = 0 \\ -1 & \text{for } t < 0 \end{cases}$
- Sinc function:  $sinc(t) = \frac{\sin(\pi t)}{\pi t}$



• Lambda function:  $\Lambda(t) = \left\{ \begin{array}{ll} 1 - |t| & \text{for } |t| \leq 1 \\ 0 & \text{for } |t| > 1 \end{array} \right]$ 



#### **Continuous Time Delta Function**

- The "function"  $\delta(t)$  is actually **not** a function.
- $\delta(t)$  is defined by the property that for all continuous functions g(t)

$$g(0) = \int_{-\infty}^{\infty} \delta(t)g(t)dt$$

• Intuitively, we may think of  $\delta(t)$  as a very short pulse with unit area.

$$g(0) = \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \left[ \frac{1}{\epsilon} \mathrm{rect}(t/\epsilon) \right] g(t) dt$$

Intuitively (but not rigorously)

$$\delta(t) = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \mathrm{rect}(t/\epsilon)$$

### **Useful CTFT Relations**

$$\delta(t) \stackrel{CTFT}{\Leftrightarrow} 1$$

$$1 \overset{CTFT}{\Leftrightarrow} \delta(f)$$

$$\mathrm{rect}(t) \stackrel{CTFT}{\Leftrightarrow} \mathrm{sinc}(f)$$

$$\mathrm{sinc}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{rect}(f)$$

$$\Lambda(t) \stackrel{CTFT}{\Leftrightarrow} \operatorname{sinc}^2(f)$$

# **CTFT Properties**

Property	Time Domain Function	CTFT
Linearity	af(t) + bg(t)	aF(f) + bG(f)
Conjugation	$f^*(t)$	$F^*(-f)$
Scaling	f(at)	$\frac{1}{ a }F(f/a)$
Shifting	$f(t-t_0)$	$\exp\left\{-j2\pi f t_0\right\} F(f)$
Modulation	$\exp\left\{j2\pi f_0 t\right\} f(t)$	$F(f-f_0)$
Convolution	f(t) * g(t)	F(f) G(f)
Multiplication	f(t)g(t)	F(f) * G(f)
Duality	F(t)	f(-f)

• Inner product property

$$\int_{-\infty}^{\infty} f(t)g^*(t)dt = \int_{-\infty}^{\infty} F(f)G^*(f)df$$