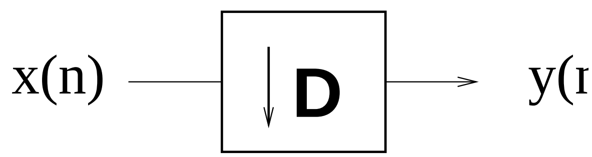
1-D Rate Conversion

- Decimation
  - Reduce the sampling rate of a discrete-time signal.
  - Low sampling rate reduces storage and computation requirements.
- Interpolation
  - Increase the sampling rate of a discrete-time signal.
  - Higher sampling rate preserves fidelity.

1-D Periodic Subsampling

- Time domain subsampling of $x(n)$ with period D

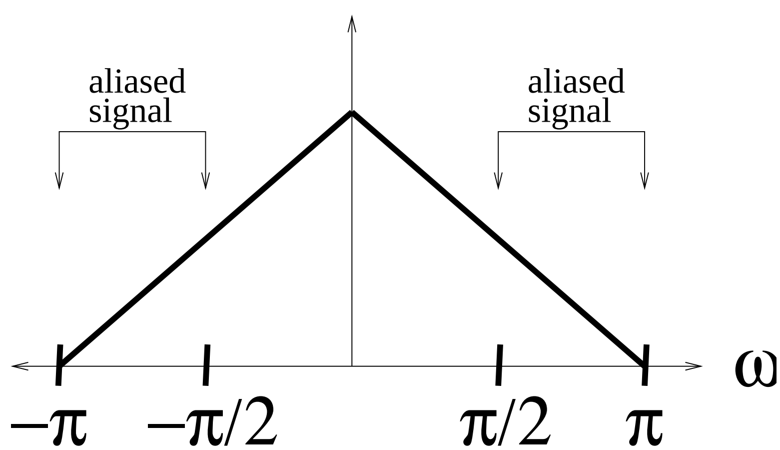
$$y(n) = x(Dn)$$



- Frequency domain representation

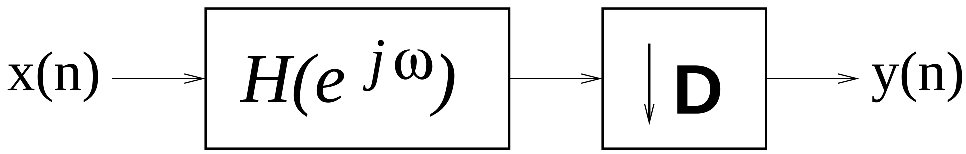
$$Y(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{j(\omega-2\pi k)/D}\right)$$

- Problem: Frequencies above π/D will alias.
- Example when $D = 2$



- Solution: Remove frequencies above π/D.

Decimation System



- Apply the filter $H(e^{j\omega})$ to remove high frequencies
  - For $|\omega| < \pi$

$$H(e^{j\omega}) = \text{rect}\left(\frac{D\omega}{2\pi}\right)$$

  - For all ω

$$\begin{aligned} H(e^{j\omega}) &= \sum_{k=-\infty}^{\infty} \text{rect}\left(D\frac{\omega - k2\pi}{2\pi}\right) \\ &= \text{prect}_{2\pi/D}(\omega) \end{aligned}$$

  - Impulse response

$$h(n) = \frac{1}{D}\text{sinc}(n/D)$$

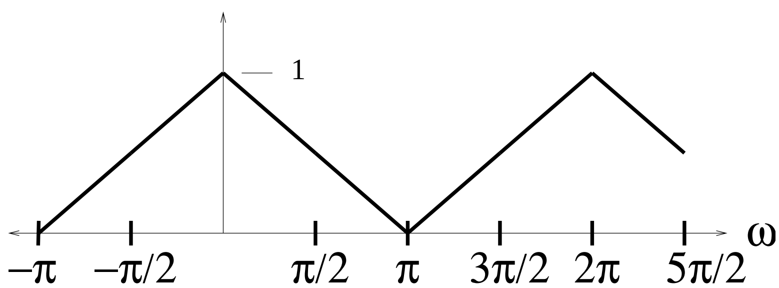
- Frequency domain representation

$$Y(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} H\left(e^{j(\omega-2\pi k)/D}\right) X\left(e^{j(\omega-2\pi k)/D}\right)$$

Graphical View of Decimation for $D = 2$

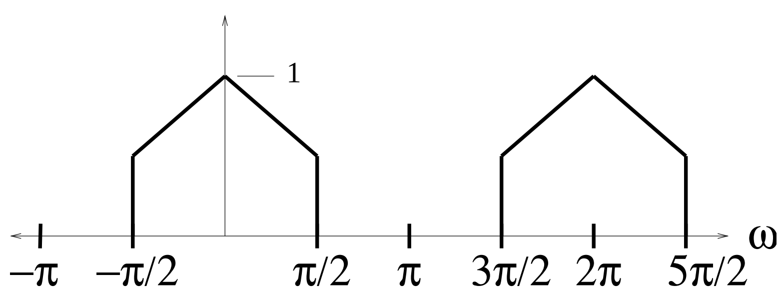
- Spectral content of signal

$$X\left(e^{j\omega}\right)$$



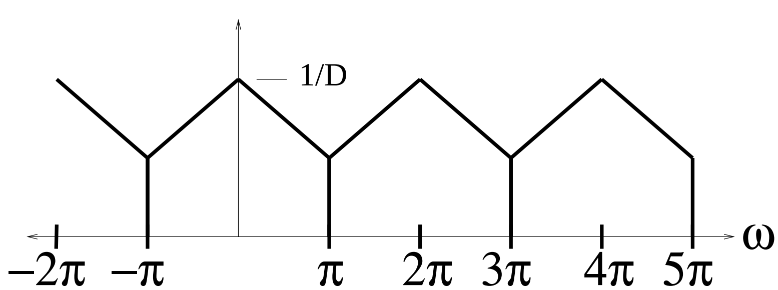
- Spectral content of filtered signal

$$H\left(e^{j\omega}\right) X\left(e^{j\omega}\right)$$



- Spectral content of decimated signal

$$Y(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} H\left(e^{j(\omega-2\pi k)/D}\right) X\left(e^{j(\omega-2\pi k)/D}\right)$$



Decimation for Images

- Extension to decimation of images is direct
- Apply 2-D Filter

$$f(i,j) = h(i,j) * x(i,j)$$

- Subsample result

$$y(i,j) = f(Di, Dj)$$

- Ideal choice of filter is

$$h(m,n) = \frac{1}{D^2}\text{sinc}(m/D)\text{sinc}(n/D)$$

- Problems:
  - Filter has infinite extent.
  - Filter is not strictly positive.

Alternative Filters for Image Decimation

- Direct subsampling

$$h(m,n) = \delta(m,n)$$

  - Advantages/Disadvantages:
    - Low computation
    - Excessive aliasing

- Block averaging

$$\begin{aligned} h(m,n) &= \delta(m,n) + \delta(m+1,n) \\ &\quad + \delta(m,n+1) + \delta(m+1,n+1) \end{aligned}$$

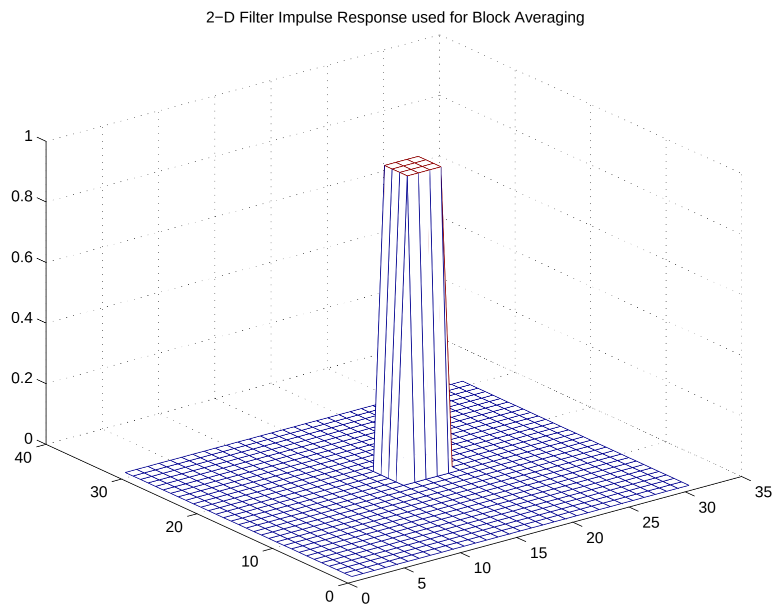
  - Advantages/Disadvantages:
    - Low computation
    - Some aliasing

- Sinc function

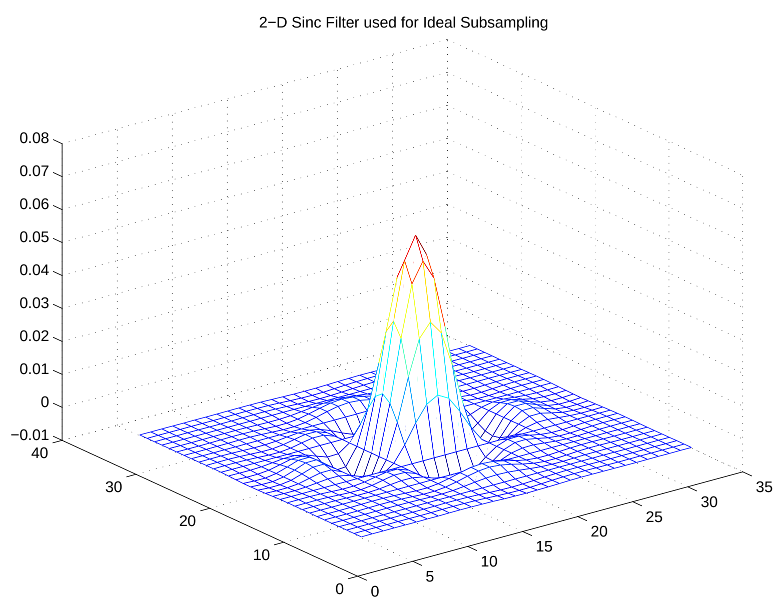
$$h(m,n) = \frac{1}{D^2}\mathrm{sinc}(m/D, n/D)$$

  - Advantages/Disadvantages:
    - Optimal, if signal is band limited...
    - High computation

Decimation Filters



Block averaging filter



Sinc filter

Original Image

- Full resolution

Image Decimation by 4 using Subsampling

- Severe aliasing

Image Decimation by 8 using Subsampling

- More severe aliasing

Image Decimation by 4 using Block Averaging

- Sharp, but with some aliasing

Image Decimation by 4 using Sinc Filter

- Theoretically optimal, but not necessarily the best visual quality

1-D Up-Sampling

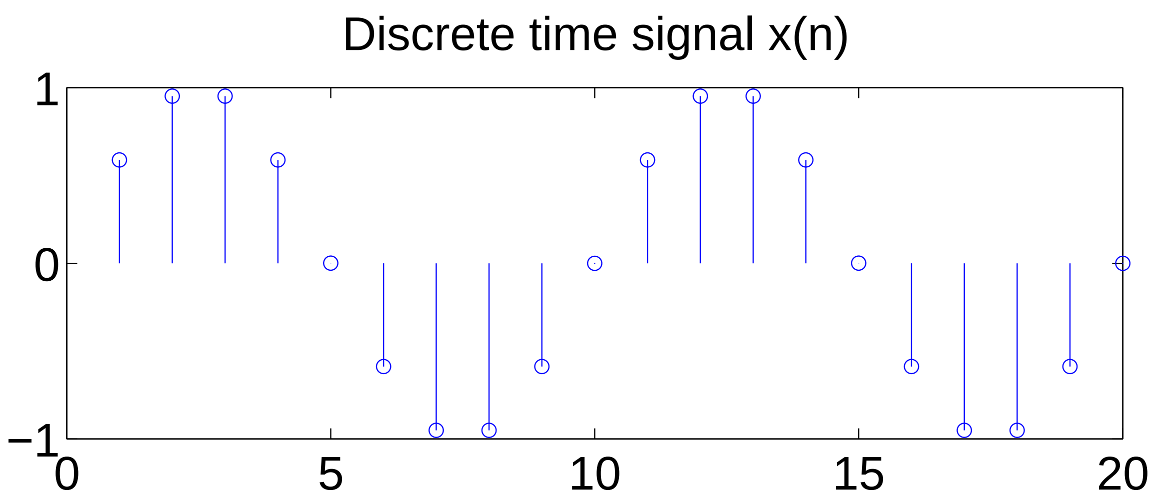
- Up-sampling by L

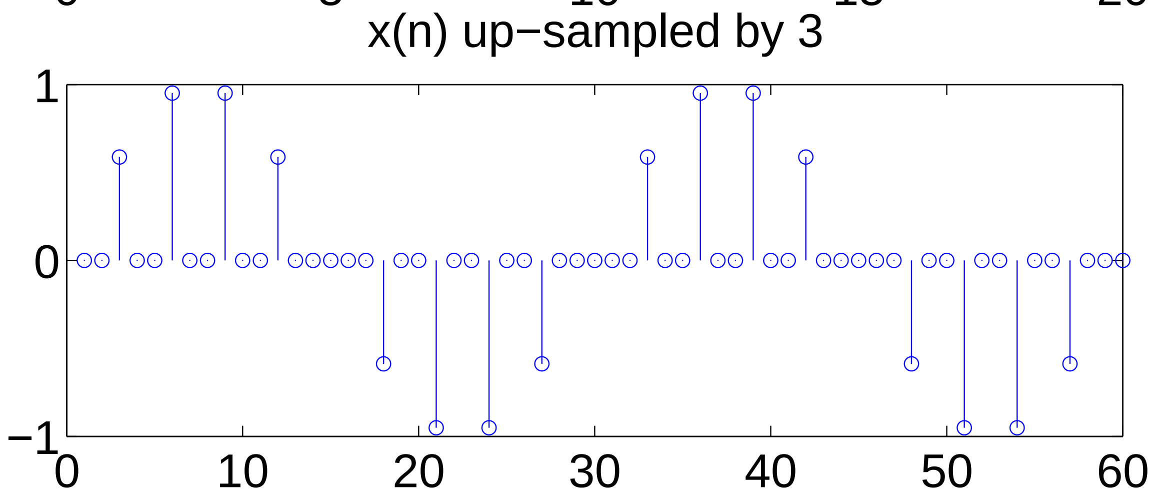
$$y(n) = \begin{cases} x(n/L) & \text{if } n = KL \text{ for some } K \\ 0 & \text{otherwise} \end{cases}$$

or the alternative form

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\delta(n - mL)$$

- Example for $L = 3$





Up-Sampling in the Frequency Domain

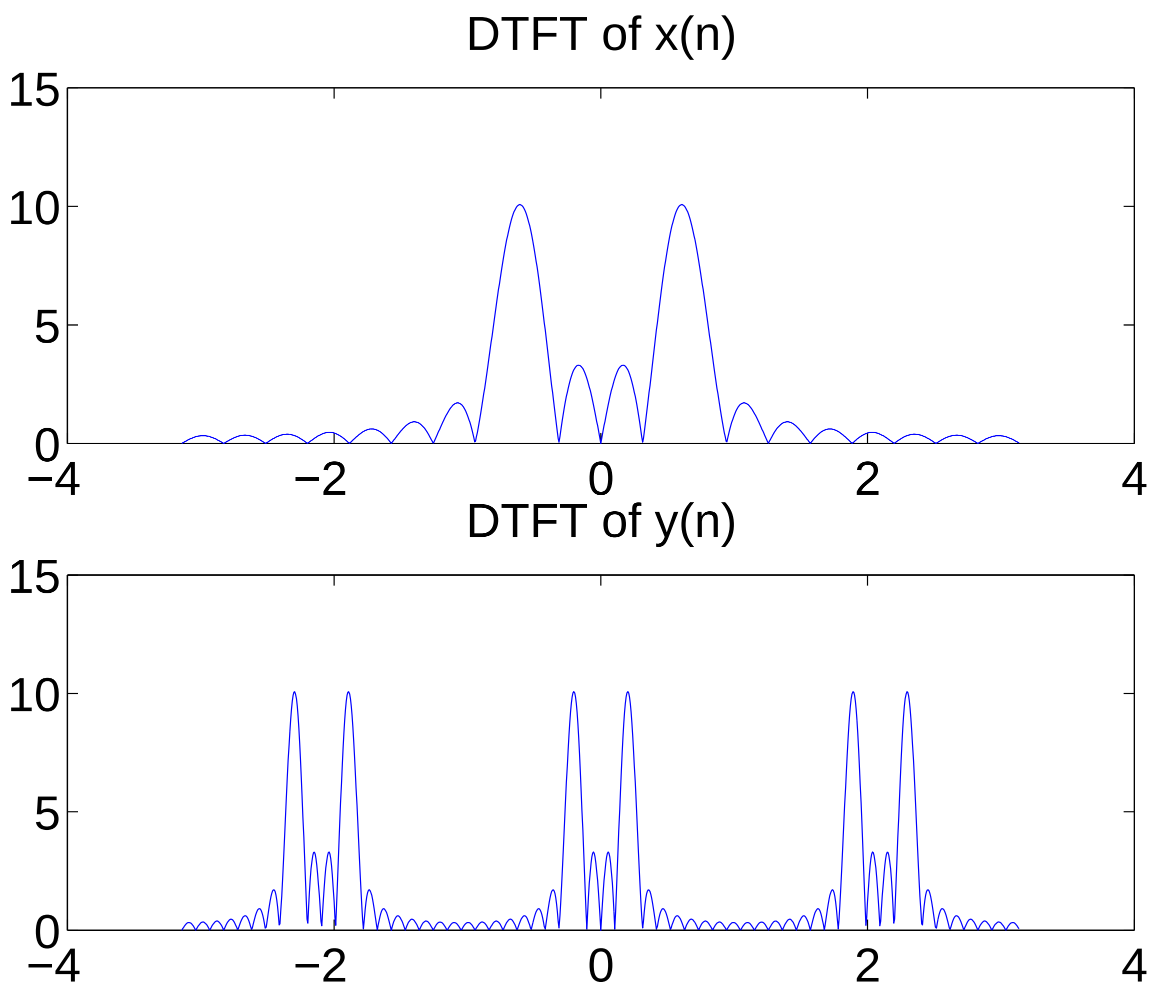
- Up-sampling by L

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\delta(n - mL)$$

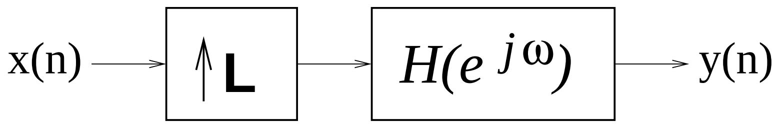
- In the frequency domain

$$Y(e^{j\omega}) = X(e^{j\omega L})$$

- Example for $L = 3$



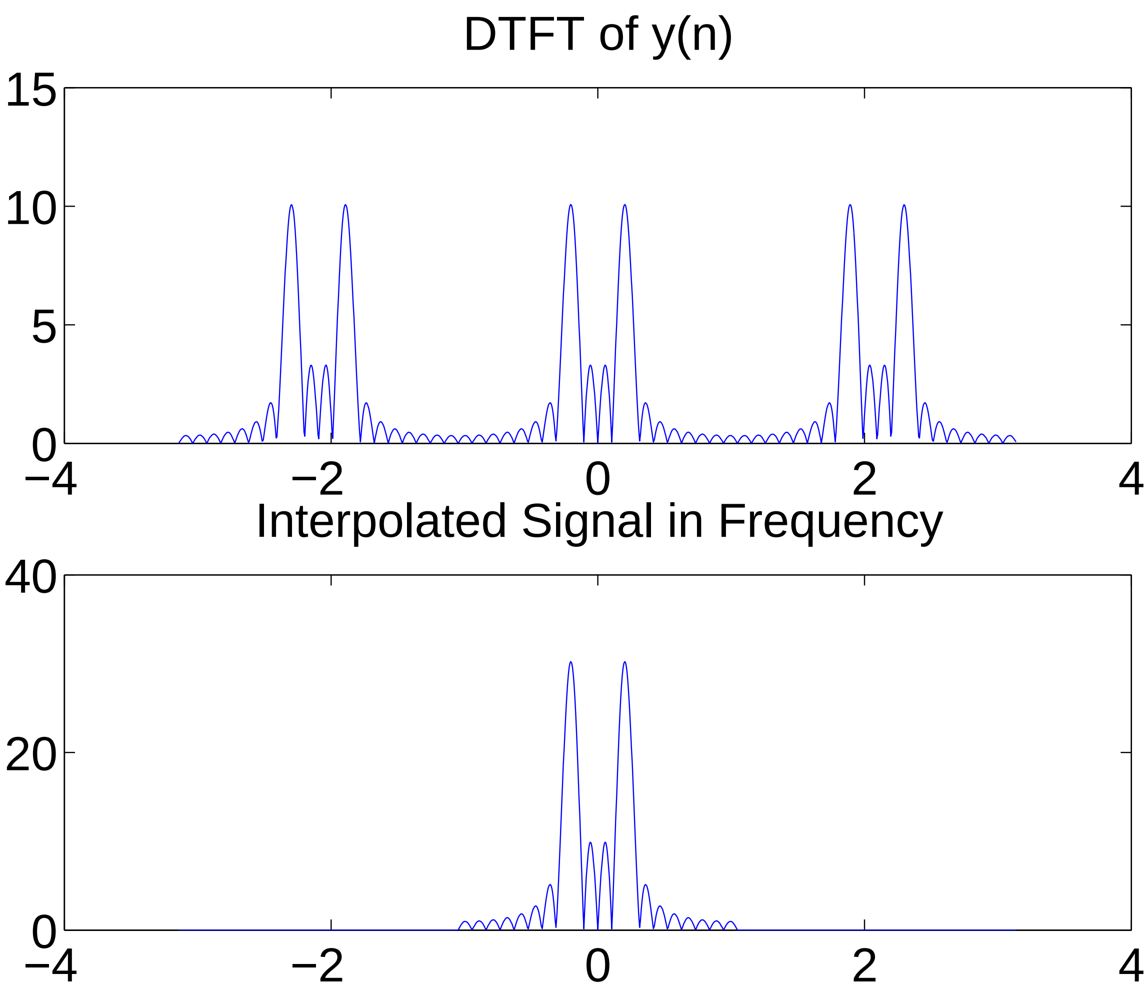
Interpolation in the Frequency Domain



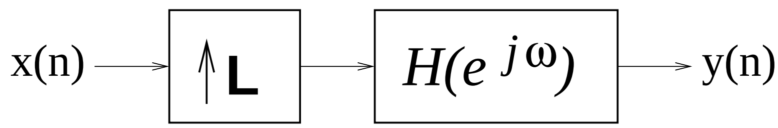
- Interpolating filter has the form

$$Y(e^{j\omega}) = H(e^{j\omega})\, X(e^{j\omega L})$$

- Example for $L = 3$



Interpolating Filter

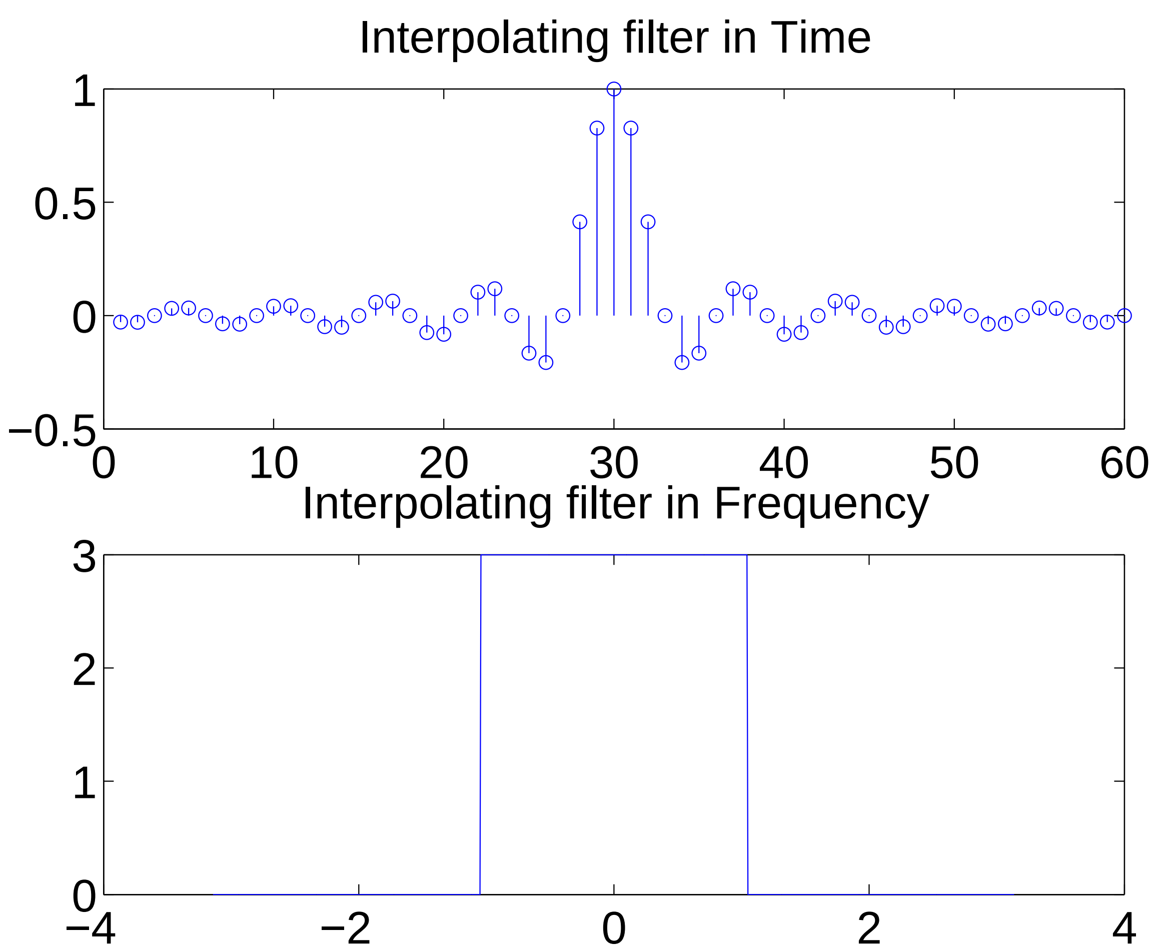


- In the frequency domain

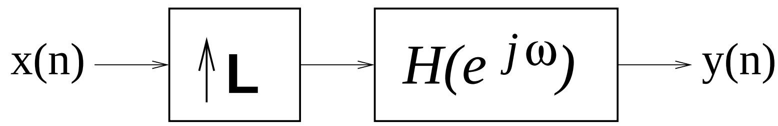
$$H(e^{j\omega}) = L\text{prect}_{2\pi/L}(\omega)$$

- In the time domain

$$h(n) = \text{sinc}\left(\frac{n}{L}\right)$$



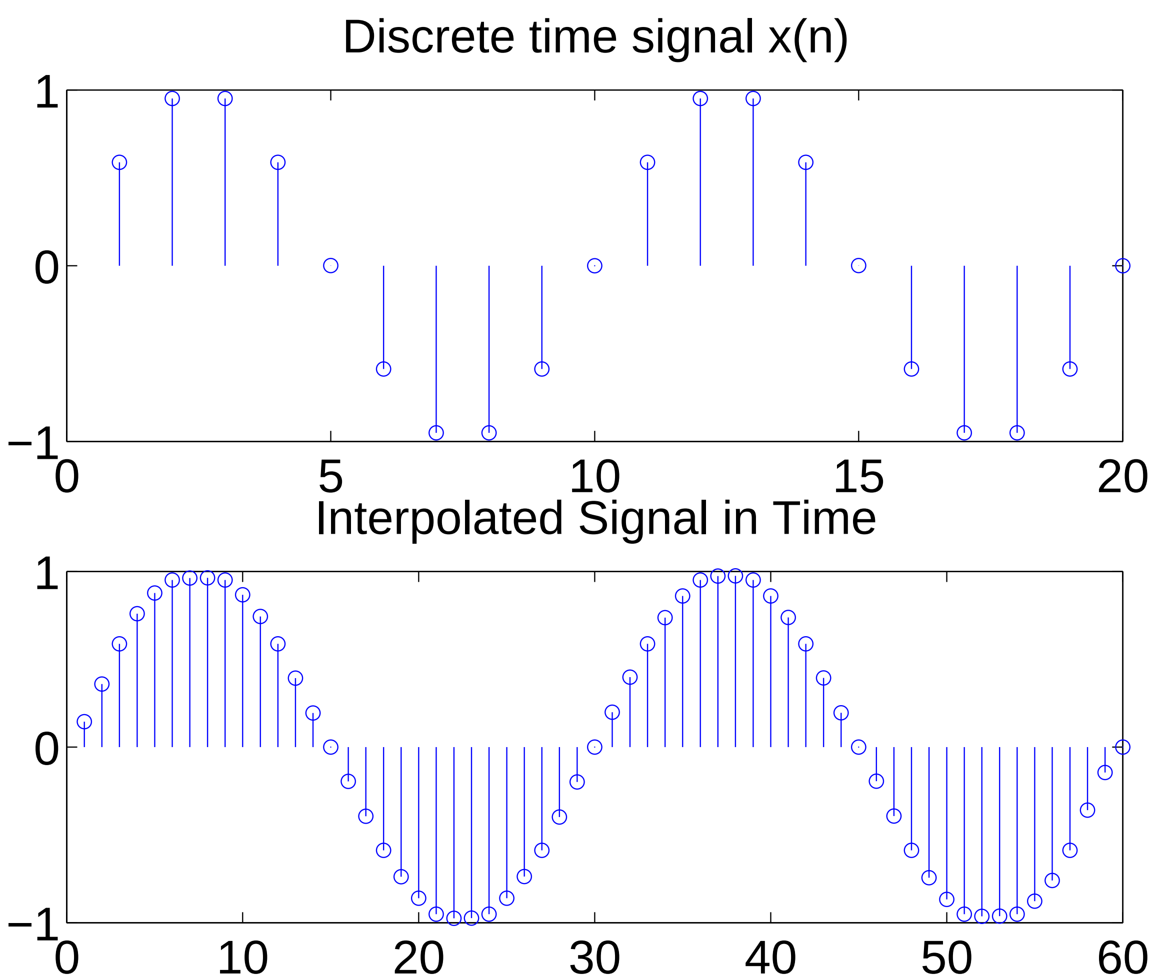
Interpolation in the Time Domain



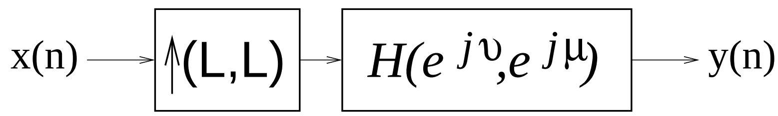
- In the frequency domain

$$Y(e^{j\omega}) = X(e^{j\omega L})$$

- Example for $L = 3$



2-D Interpolation



- Up-sampling

$$y(m,n) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} x(k,l)\delta(m-kL, n-lL)$$

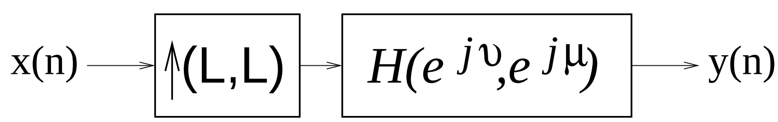
- In the frequency domain

$$H(e^{j\mu}, e^{j\nu}) = \text{prect}_{2\pi/L}(\mu)\text{prect}_{2\pi/L}(\nu)$$

$$h(m,n) = \text{sinc}\left(m/L, n/L\right)$$

- Problems: sinc function impulse response
  - Infinite support; infinite computation
  - Negative sidelobes; ringing artifacts at edges

Pixel Replication

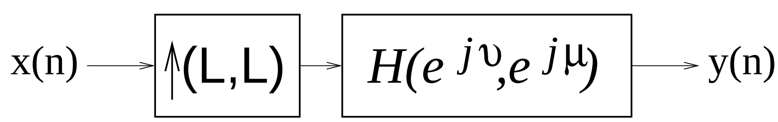


- Impulse response of filter

$$h(m,n) = \begin{cases} 1 & \text{for } 0 \leq m \leq L-1 \text{ and } 0 \leq n \leq L-1 \\ 0 & \text{otherwise} \end{cases}$$

- Replicates each pixel L^2 times

Bilinear Interpolation



- Impulse response of filter

$$h(m, n) = \Lambda(m/L)\Lambda(n/L)$$

- Results in linear interpolation of intermediate pixels