EE 637 Study Solutions - Assignment 7

Topics: Color matching, additive and subtractive color

Spring 2010 Exam 2: Problem 2 (color and gamma)

Consider the color display that produces the following color (X, Y, Z) when given the input is (r, g, b).

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (r/255)^\alpha \\ (g/255)^\alpha \\ (b/255)^\alpha \end{bmatrix}$$

a) What is the gamma of the display?

Solution:

$$\gamma = \alpha$$

b) What is the white point of the display?

Solution:

$$\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow (x_w, y_w) = \left(\frac{1}{3}, \frac{1}{3} \right)$$

$$\Rightarrow \text{ Equal energy white}$$

c) What are the color primaries of the display?

Solution:

$$(X_r, Y_r, Z_r) = (1, 00)$$

$$(x_r, y_r) = (1, 0)$$

$$(x_g, y_g) = (0, 1)$$

$$(x_b, y_b) = (0, 0)$$

d) Can such a display be physically built? Why or why not?

Solution:

No, because these primary colors do not exist physically.

e) You produce two images, one with a checker board pattern of values 0 and 255, and a second with a constant value of $(r, g, b) = (a, a, a)$. The value of a is then adjusted so that the two images match when viewed from a distance. What is the value of gamma for the display in terms of the value a?

Solution:

Image 1: the average color is

$$(X, Y, Z) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

Image 2: the average color is

$$\begin{aligned} (X, Y, Z) &= (1, 1, 1)\left(\frac{a}{255}\right)^{\alpha} \\ \Rightarrow \left(\frac{a}{255}\right)^{\alpha} &= \frac{1}{2} \\ \alpha \log\left(\frac{a}{255}\right) &= \log\left(\frac{1}{2}\right) \\ \alpha &= \frac{\log\left(\frac{1}{2}\right)}{\log\left(\frac{a}{255}\right)} \end{aligned}$$

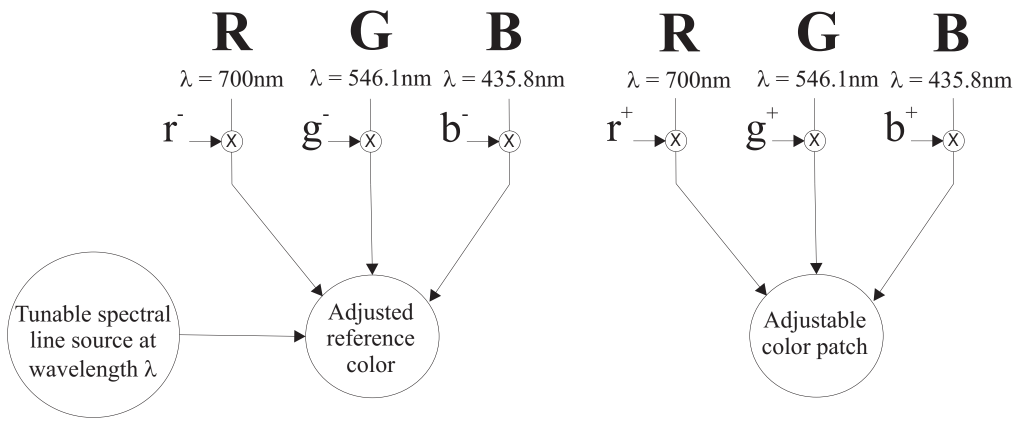
f) Imagine that $\alpha = 1$ and the values of (r, g, b) are quantized to 8-bits. Describe the defects you would expect to see in the displayed image.

Solution:

You would see quantization artifacts in dark regions of the image.

Spring 2006 Exam 2: Problem 2 (color matching)

Consider the color matching experiment represented by the following diagram.



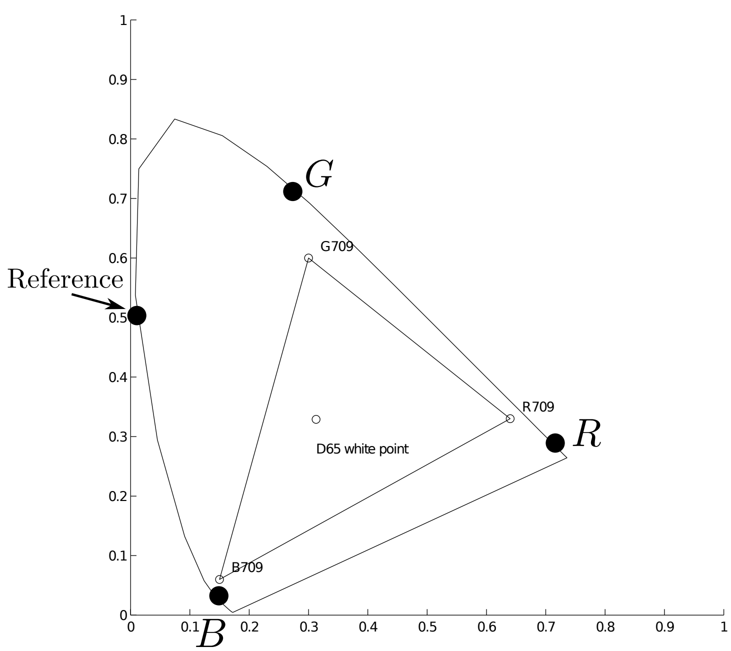
a) Why is it necessary to have both the values of (r^+, g^+, b^+) and (r^-, g^-, b^-) for this matching experiment?

Solution:

In general, the reference patches cannot be matched with only (r^+, g^+, b^+). Therefore, color must also be subtracted from the reference patch using (r^-, g^-, b^-).

b) For this part, assume that the reference spectral line source to be matched before adjustment is at 480 nm. Indicate the approximate location of the primary colors and the reference line source on the following chromaticity diagram.

Solution:



c) For this part, assume that the reference spectral line source to be matched before adjustment is at 480 nm. Which values of (r^+, g^+, b^+) and (r^-, g^-, b^-) will be 0, positive, or negative?

Solution:

$$
\begin{aligned}
r^+ &= 0 & r^- &> 0 \\
g^+ &> 0 & g^- &= 0 \\
b^+ &> 0 & b^- &= 0
\end{aligned}
$$

d) Specify how one would measure the color matching functions $r(\lambda)$, $g(\lambda)$, $b(\lambda)$ for the three primaries given in this experiment?

Solution:

Vary λ from 400nm to 700nm and set

$$
\begin{aligned}
r(\lambda) &= r^+ - r^- \\
g(\lambda) &= g^+ - g^- \\
b(\lambda) &= b^+ - b^-
\end{aligned}
$$

Spring 2003 Exam 2: Problem 3 (subtractive color)

In the following problem, we assume that spectral light measurements are discretized into 31 component vectors ranging from 400 nm to 700 nm in 10 nm steps. Using this assumption, the light reflected from an object has the spectrum

$$I_i = R_i S_i$$

where $1 \leq i \leq 31$ and S_i is the source illumination, R_i is the surface reflectance, and I_i is the reflected light. Further define, x_i, y_i, and z_i as the color matching functions for the X, Y, Z tristimulus values.

For documents printed on a *PurdueJet* printer, it is known that the spectral surface reflectance is given by

$$R = \begin{bmatrix} R_1 \\ \vdots \\ R_{31} \end{bmatrix} = \mathbf{1} - \mathbf{A} \begin{bmatrix} c \\ m \\ y \end{bmatrix}$$

where the columns of the matrix $\mathbf{A}$ are the spectral absorptance's of the cyan, magenta, and yellow inks respectively, and $\mathbf{1}$ is the 31 dimensional column vector $[1, \cdots, 1]^t$.

a) Calculate an equation for the X, Y, Z components of the reflected light in terms of I_i.

Solution:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \begin{bmatrix} I_i \end{bmatrix} = \begin{bmatrix} \sum_i x_i I_i \\ \sum_i y_i I_i \\ \sum_i z_i I_i \end{bmatrix}$$

Define the matrix

$$B_{i,j} = \begin{cases} x_j & \text{for } i = 1 \\ y_j & \text{for } i = 2 \\ z_j & \text{for } i = 3 \end{cases}$$

and the vector

$$I = \begin{bmatrix} I_1 \\ \vdots \\ I_{31} \end{bmatrix}$$

then

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = BI$$

b) In general, is it possible for two different surface reflectance functions R' and R'' to have the same X, Y, Z components? Characterize the space of possible spectral differences $\Delta R = R'' - R'$ that will result in no change of the X, Y, Z components.

Solution:

Yes. Define

$$D = \text{diag}\{S_1, \ldots, S_{31}\}$$

Then

$$I = DR$$

Where

$$R = \begin{bmatrix} R_1 \\ \vdots \\ R_{31} \end{bmatrix}$$

So we know that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = BDR$$

(BD) is a 3x31 matrix.

So we know that

$$BD\Delta R = 0$$

$$\Rightarrow \Delta R \text{ is in the null space of } (BD)$$

$$\Rightarrow \Delta R \text{ falls in a 28 dimensional subspace}$$

c) For documents printed on a *PurdueJet* printer, calculate an expression for the vector $[c, m, y]^t$ as a function of the measured value of $[X, Y, Z]^t$ and the known illuminant S. (Hint: You will need to define matrices in terms of the color matching functions and the known illuminant.)

Solution:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = BDR$$

$$= BDA \begin{bmatrix} c \\ m \\ y \end{bmatrix}$$

$$\begin{bmatrix} c \\ m \\ y \end{bmatrix} = [BDA]^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

BDA is a 3x3 matrix

d) For documents printed on a *PurdueJet* printer, calculate an expression for the spectral reflectance vector R as a function of the measured value of $[X, Y, Z]^t$ and the known illuminant S.

Solution:

$$R = A \begin{bmatrix} c \\ m \\ y \end{bmatrix} = A\,[BDA]^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$