Topics: Digital Halftoning

Spring 2010 Final: Problem 3 (quality metrics for halftoned images)

Assume that for achromatic images, you use the following fidelity metric for the human visual system,

$$D = \sum_{m,n} \left(h(m,n) * b(m,n) - h(m,n) * g(m,n)\right)^2$$

where D is a measure of distortion between the linear gray scale image $g(m,n)$ and the binary image $b(m,n)$, and $*$ indicates 2D convolution.

Furthermore, assume that

$$h(m,n) = [\delta(m,n) + (1/2)\,(\delta(m-1,n) + \delta(m+1,n))]*[\delta(m,n) + (1/2)\,(\delta(m,n-1) + \delta(m,n+1))]$$

where $*$ represents 2D convolution.

a) Calculate the DSFT, $H(e^{j\mu}, e^{j\nu})$ of $h(m,n)$, and sketch its shape.

Solution:

$$\delta(n) + \frac{1}{2}\left(\delta(n-1) + \delta(n+1)\right) \overset{DTFT}{\Longleftrightarrow} 1 + \frac{1}{2}\left(e^{-j\omega} + e^{j\omega}\right) = 1 + \cos(\omega)$$

$$h(m,n) \overset{DSFT}{\Longleftrightarrow} (1 + \cos(\mu))\,(1 + \cos(\nu)) = H\left(e^{j\mu}, e^{j\nu}\right)$$

b) What are the values of $H(e^{j0}, e^{j0})$, $H(e^{j\pi}, e^{j0})$, $H(e^{-j\pi}, e^{j0})$, $H(e^{j0}, e^{j\pi})$, and $H(e^{j0}, e^{-j\pi})$.

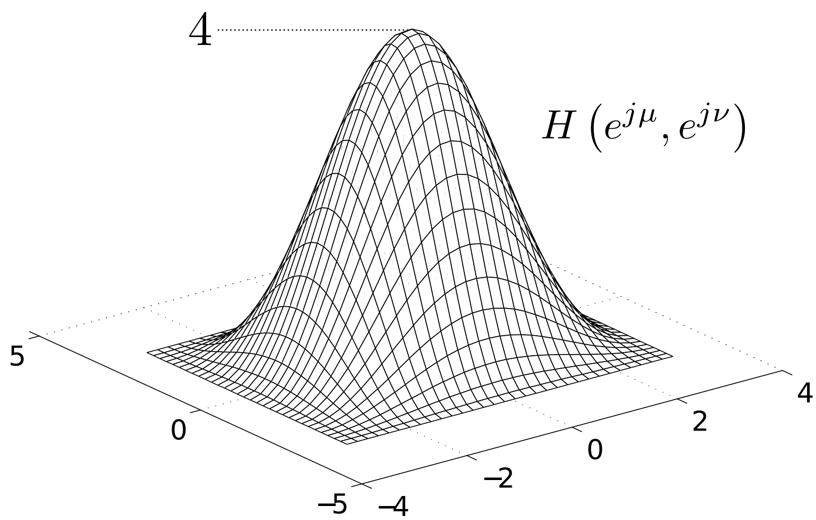
Solution:

$$H(e^{j0}, e^{j0}) = 4$$
$$H(e^{j\pi}, e^{j0}) = H(e^{-j\pi}, e^{j0}) = 0(2) = 0$$
$$H(e^{j0}, e^{j\pi}) = H(e^{j0}, e^{-j\pi}) = 2(0) = 0$$

c) Assuming your objective is to represent the gray scale image $g(m,n)$ by the binary image $b(m,n)$, then is it best for $d(m,n) = b(m,n) - g(m,n)$ to contain mostly high frequencies or low frequencies? Why?

Solution:

It is best to push error to high frequencies, where they are more difficult to see.



d) If $g(m,n) = 1/2$, then determine all binary patterns $b(m,n)$ (i.e. a pattern of 1's and 0's) that best matches $g(m,n)$.

Solution:

The following binary patterns result in no low frequency error and produce $D = 0$:

$$b(m,n) = \frac{(-1)^m(-1)^n + 1}{2}$$

or

$$b(m,n) = \frac{(-1)^m + 1}{2}$$

or

$$b(m,n) = \frac{(-1)^n + 1}{2}$$

e) How could the distortion measure, D, be improved to better account for contrast?

Solution:

$$D' = \sum_{m,n} \left((h(m,n) * b(m,n))^{\frac{1}{3}} - (h(m,n) * g(m,n))^{\frac{1}{3}} \right)^2$$

The power 1/3 accounts for sensitivity to contrast.

Spring 2009 Final: Problem 5 (error diffusion)

Consider the 1-D error diffusion algorithm specified by the equations

$$\begin{aligned} b_n &= Q(y_n) \\ e_n &= y_n - b_n \\ y_n &= x_n + e_{n-1} \end{aligned}$$

where x_n is the input, b_n is the output, and $Q(\cdot)$ is a binary quantizer with the form

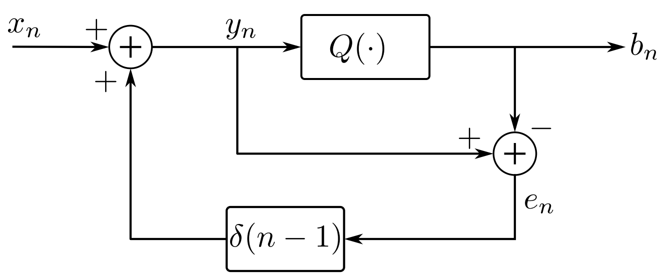
$$Q(y) = \begin{cases} 1 & \text{if } y > 0.5 \\ 0 & \text{if } y \leq 0.5 \end{cases} .$$

where we assume that $e_0 = 0$ and the algorithm is run for $n \geq 1$.

Furthermore, define $d_n = x_n - b_n$.

a) Draw a flow diagram for this algorithm. Make sure to label all the signals in the flow diagram using the notation defined above.

Solution:



b) Calculate b_n for $n = 1$ to 10 when $x_n = 0.25$ and $e_0 = 0$.

Solution:

n	1	2	3	4	5	6	7	8	9	10
x_n	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
y_n	0.25	0.5	0.75	0	0.25	0.5	0.75	0	0.25	0.5
b_n	0	0	1	0	0	0	1	0	0	0
e_n	0.25	0.5	−0.25	0	0.25	0.5	−0.25	0	0.25	0.5

c) Calculate an expression for d_n in terms of the quantization error e_n.

Solution:

$$d_n = x_n - b_n = x_n - (y_n - e_n) = x_n - (x_n + e_{n-1} - e_n) = e_n - e_{n-1}$$

d) Calculate an expression for $\sum_{n=1}^{N} d_n$ in terms of the quantization error e_n.

Solution:

$$\sum_{n=1}^{N} d_n = \sum_{n=1}^{N} (e_n - e_{n-1}) = e_N - e_0 = e_N$$

e) What does the result of part d) above tell you about the output of error diffusion?

Solution:

1) It tells us that the accumulated error is bounded. Now we prove that e_N is bounded.

   i) When $N = 0$, $e_0 = 0$ is bounded.

   ii) We assume that e_k is bounded, then $y_{k+1} = x_{k+1} + e_k$ is bounded since x_{k+1} is bounded as well. Then $e_{k+1} = y_{k+1} - b_{k+1}$ is bounded since y_{k+1} is bounded and $b_{k+1} = Q(y_{k+1})$ is bounded. Thus e_k is bounded and e_{k+1} is bounded.

2) It tells us that the total average of the signal is approximately maintained.

$$\begin{aligned} \sum_{n=1}^{N} d_n &= \sum_{n=1}^{N} (x_n - b_n) = e_N \\ \Rightarrow \sum_{n=1}^{N} b_n &= \sum_{n=1}^{N} x_n - e_N \\ \Rightarrow \frac{1}{N} \sum_{n=1}^{N} b_n &= \frac{1}{N} \sum_{n=1}^{N} x_n - \frac{1}{N} e_N \end{aligned}$$

So the local average of the output is approximately the same as the local average of the input.

Spring 2004 Final: Problem 2 (halftoning/power spectrum)

Let $X(m,n)$ be an achromatic image taking continous values in the interval $[0,1]$, and let $T(m,n)$ be a 2-D random field of i.i.d. random variables which are uniformly distributed on the interval $[0,1]$. Let the halftoned version of $X(m,n)$ be given by

$$Y(m,n) = \begin{cases} 1 & \text{if } X(m,n) \geq T(m,n) \\ 0 & \text{if } X(m,n) < T(m,n) \end{cases}$$

a) Is $Y(m,n)$ a stationary random process?

Solution:

No, because $E[Y(m,n)]$ is a function of (m,n).

b) Calculate $\mu(m,n) = E[Y(m,n)]$

Solution:

$$\begin{aligned} E[Y(m,n)] &= 1 \cdot P\{T(m,n) \leq X(m,n)\} \\ &= X(m,n) \end{aligned}$$

c) Calculate $E[D(m,n)D(m+k,n+l)]$ for $D(m,n) = Y(m,n) - \mu(m,n)$.

Solution:

$$E[D(m,n)D(m+k,n+l)] = \delta(k,l)E\left[(D(m,n))^2\right]$$

$$= \delta(k,l)\left[(1-X(m,n))^2 P\{T(m,n) \leq X(m,n)\} + (-X(m,n))^2 P\{T(m,n) > X(m,n)\}\right]$$

$$= \delta(k,l)\left[(1-X(m,n))^2 X(m,n) + (X(m,n))^2 (1-X(m,n))\right]$$

$$= \delta(k,l)(1-X(m,n))(X(m,n))\left[1-X(m,n)+X(m,n)\right]$$

$$= \delta(k,l)(1-X(m,n))(X(m,n))$$

d) Is $D(m,n)$ a stationary random process?

Solution:

No, because $E\left[(D(m,n))^2\right]$ is a function of (m,n).

e) For the special case of $X(m,n) = g$, compute the power spectral density $S(e^{j\mu}, e^{j\nu})$ of $D(m,n)$.

Solution:

In this case,

$$R(k,l) = \delta(k,l)(1-g)g$$

$$S\left(e^{j\mu}, e^{j\nu}\right) = (1-g)g$$

f) Is $Y(m,n)$ a good quality halftone of $X(m,n)$. Justify your answer.

Solution:

No, because it contains too much low frequency energy.