EE 637 Midterm I

February 20, Spring 2015

**Name:** (4 pt) ____________________________________

**Instructions:**

- This is a 50 minute exam containing **three** problems.
- You may **only** use your brain and a pencil (or pen) and the included "Fact Sheet" to complete this exam.
- You **may not** use or have access to your book, notes, any supplementary reference, a calculator, or any communication device including a cell-phone or computer.
- You **may not** communicate with any person other than the official proctor during the exam.

# Good Luck.

# Fact Sheet

- Function definitions

$$\mathrm{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1-|t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFT

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \\ x(t) &= \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \end{aligned}$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X(-f)$$

$$x(t-t_0) \overset{CTFT}{\Leftrightarrow} X(f) e^{-j2\pi f t_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|} X(f/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} x(-f)$$

$$x(t) e^{j2\pi f_0 t} \overset{CTFT}{\Leftrightarrow} X(f-f_0)$$

$$x(t) y(t) \overset{CTFT}{\Leftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(f) Y(f)$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) Y^*(f) df$$

- CTFT pairs

$$\mathrm{sinc}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{rect}(f)$$

$$\mathrm{rect}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{sinc}(f)$$

For $a > 0$

$$\frac{1}{(n-1)!} t^{n-1} e^{-at} u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{(j2\pi f + a)^n}$$

- CSFT

$$\begin{aligned} F(u,v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx dy \\ f(x,y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} du dv \end{aligned}$$

- DTFT

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \end{aligned}$$

- DTFT pairs

$$a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{1 - a e^{-j\omega}}$$

- Rep and Comb relations

$$\mathrm{rep}_T\left[x(t)\right] = \sum_{k=-\infty}^{\infty} x(t-kT)$$

$$\mathrm{comb}_T\left[x(t)\right] = x(t) \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$\mathrm{comb}_T\left[x(t)\right] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \mathrm{rep}_{\frac{1}{T}}\left[X(f)\right]$$

$$\mathrm{rep}_T\left[x(t)\right] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \mathrm{comb}_{\frac{1}{T}}\left[X(f)\right]$$

- Sampling and Reconstruction

$$\begin{aligned} y(n) &= x(nT) \\ Y(e^{j\omega}) &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right) \\ s(t) &= \sum_{k=-\infty}^{\infty} y(k) \delta(t-kT) \\ S(e^{j\omega}) &= Y\left(e^{j\omega T}\right) \end{aligned}$$

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**Problem 1.**(32pt)

Consider the following 1D system with input $x(n)$ and output $y(n)$.

$$y(n) = x(n) + \lambda \left( x(n) - \frac{1}{3} \sum_{k=-1}^{1} x(n-k) \right) .$$

a) Is this a linear system? Is this a space invariant system?
b) Calculate and sketch the impulse response, $h(n)$ for $\lambda = 0.5$.
c) Calculate and sketch the frequency response, $H(e^{j\omega})$ for $\lambda = 0.5$.
d) Describe how the filter behaves when $\lambda$ is positive and large.

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**Problem 2.**(32pt)

Consider the 2D discrete space signal $x(m,n)$ with the DSFT of $X(e^{j\mu}, e^{j\nu})$ given by

$$X(e^{j\mu}, e^{j\nu}) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(m,n) e^{-j(m\mu+n\nu)} \ .$$

Then define

$$\begin{aligned} p_0(n) &= \sum_{m=-\infty}^{\infty} x(m,n) \\ p_1(m) &= \sum_{n=-\infty}^{\infty} x(m,n) \end{aligned}$$

with corresponding DTFT given by

$$\begin{aligned} P_0(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} p_0(n) e^{-jn\omega} \\ P_1(e^{j\omega}) &= \sum_{m=-\infty}^{\infty} p_1(m) e^{-jm\omega} \end{aligned}$$

a) Derive an expression for $P_0(e^{j\omega})$ in terms of $X(e^{j\mu}, e^{j\nu})$.

b) Derive an expression for $P_1(e^{j\omega})$ in terms of $X(e^{j\mu}, e^{j\nu})$.

c) Find a function $x(m,n)$ that is **not zero everywhere** such that $p_0(n) = p_1(m) = 0$ for all $m$ and $n$.

d) Do the functions $p_0(n)$ and $p_1(m)$ together contain sufficient information to uniquely reconstruct the function $x(m,n)$? Justify your answer.

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**Problem 3.**(32pt)

Let $x(t) = \text{sinc}(t/a)$ for some positive constant $a$, and let $y(n) = x(nT)$ where $f_s = 1/T$ is the sampling frequency of the system. Further assume that $a$ has units of $sec$, $T$ has units of $sec$, and $f_s$ has units of $Hz = sec^{-1}$.

a) Calculate and sketch $X(f)$, the CTFT of $x(t)$.

b) Calculate $Y(e^{j\omega})$, the DTFT of $y(n)$.

c) What is the minimum sampling frequency, $f_s$, that ensures perfect reconstruction of the signal?

d) Sketch the function $Y(e^{j\omega})$ on the interval $[-\pi, \pi]$ when $T = a$.

e) Sketch the function $Y(e^{j\omega})$ on the interval $[-\pi, \pi]$ when $T = \frac{5a}{4}$.

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