Topics: Chromaticity, white point, and quality metrics

Spring 2010 Final: Problem 2 (Lab color transform)

The approximate Lab color space transform is given by

$$\begin{aligned} L &= 100(Y/Y_0)^{1/3} \\ a &= 500\left[(X/X_0)^{1/3} - (Y/Y_0)^{1/3}\right] \\ b &= 200\left[(Y/Y_0)^{1/3} - (Z/Z_0)^{1/3}\right] \end{aligned}$$

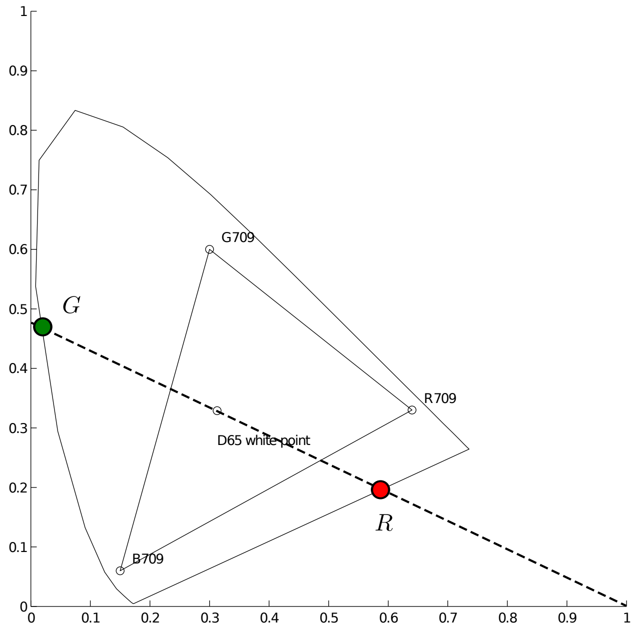
a) Qualitatively specify the colors corresponding to the following values of L, a, b: 1) $L = 100$, $a = 0$, $b = 0$; 2) $L = 100$, $a =$ large positive, $b = 0$; 3) $L = 100$, $a = 0$, $b =$ large positive; 4) $L = 100$, $a =$ large negative, $b = 0$; and 5) $L = 100$, $a = 0$, $b =$ large negative;

Solution:

$$\begin{aligned} (100, 0, 0) &\Rightarrow \text{Bright white corresponding to } (X_0, Y_0, Z_0) \\ (100, \text{large positive}, 0) &\Rightarrow \text{saturated red (opponent to green)} \\ (100, 0, \text{large positive}) &\Rightarrow \text{saturated yellow (opponent to blue)} \\ (100, \text{large negative}, 0) &\Rightarrow \text{saturated green (opponent to red)} \\ (100, 0, \text{large negative}) &\Rightarrow \text{saturated blue (opponent to yellow)} \end{aligned}$$

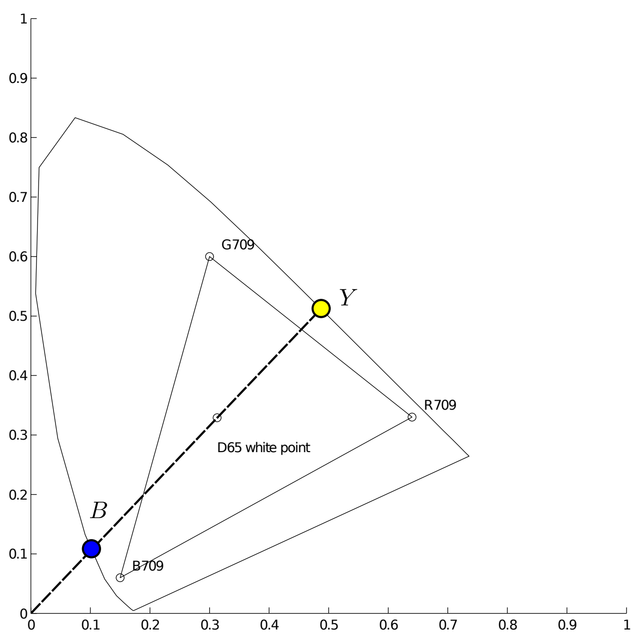
For $(L, a, b) = (100, a, 0)$, it must be that $Y = Y_0$ and $Z = Z_0$, so that the color's (X, Y, Z) components must be given by $(X, Y, Z) = (X_0, Y_0, Z_0) + (X, 0, 0)$ for some value of X. This means that the chromaticity of the color (X, Y, Z) must fall on a line connecting $\boldsymbol{W} = (X_0, Y_0, Z_0)$ and $\boldsymbol{X} = (X, 0, 0)$, where $\boldsymbol{W}$ is the white point of the L, a, b transform and $\boldsymbol{X}$ is the imaginary primary for the X component of the (X, Y, Z) coordinate system.

This line between $\boldsymbol{W}$ and $\boldsymbol{X}$ is illustrated in the following figure. The resulting color can then be expressed as Color$= \boldsymbol{W} + \alpha\boldsymbol{Z}$, where α is a constant. When $a = 0$ and Color$= \boldsymbol{W}$, it must be that $\alpha = 0$. When $\alpha > 0$, then Color$= \boldsymbol{W} + \alpha\boldsymbol{X}$, where $\alpha > 0$. When $a < 0$, then Color$= \boldsymbol{W} + \alpha\boldsymbol{X}$, where $\alpha < 0$. The largest value of that results in a real color that is indicated by the point $\boldsymbol{R}$ on the diagram, and the smallest value results in a real color that is indicated by the point $\boldsymbol{G}$ on the diagram.



For $(L, a, b) = (100, 0, b)$, the situation is similar. In this case, the chromaticity of the color (X, Y, Z) must fall on a line connecting $\boldsymbol{W}$ and $\boldsymbol{Z} = (0, 0, Z)$, where $\boldsymbol{Z}$ is the imaginary primary for the Z component of the (X, Y, Z) coordinate system.

This line between $\boldsymbol{W}$ and $\boldsymbol{Z}$ is illustrated below. The resulting color can then be expressed as Color$= \boldsymbol{W} + \beta\boldsymbol{Z}$, where β is a constant. When $b = 0$, then Color$= \boldsymbol{W}$, and it must be that $\beta = 0$. When $b < 0$, then Color$= \boldsymbol{W} + \beta\boldsymbol{Z}$, where $\beta > 0$. When $b > 0$, then Color$= \boldsymbol{W} + \beta\boldsymbol{Z}$, where $\beta < 0$. The largest value of b that results in a real color is indicated by the point $\boldsymbol{Y}$ on the diagram, and the smallest value of b that results in a real color is indicated but the point $\boldsymbol{B}$ on the diagram.



b) Which component of the L, a, b coordinate system requires the greatest spatial frequency to accurately represent? Why?

Solution:

L requires the greatest spatial frequency because the contrast sensitivity falls off most slowly in luminance.

c) Is the L, a, b coordinate system suitable for representing images that will be JPEG compressed? Why or why not?

Solution:

No, because it is designed for color matching of large patches corresponding to low spatial frequency.

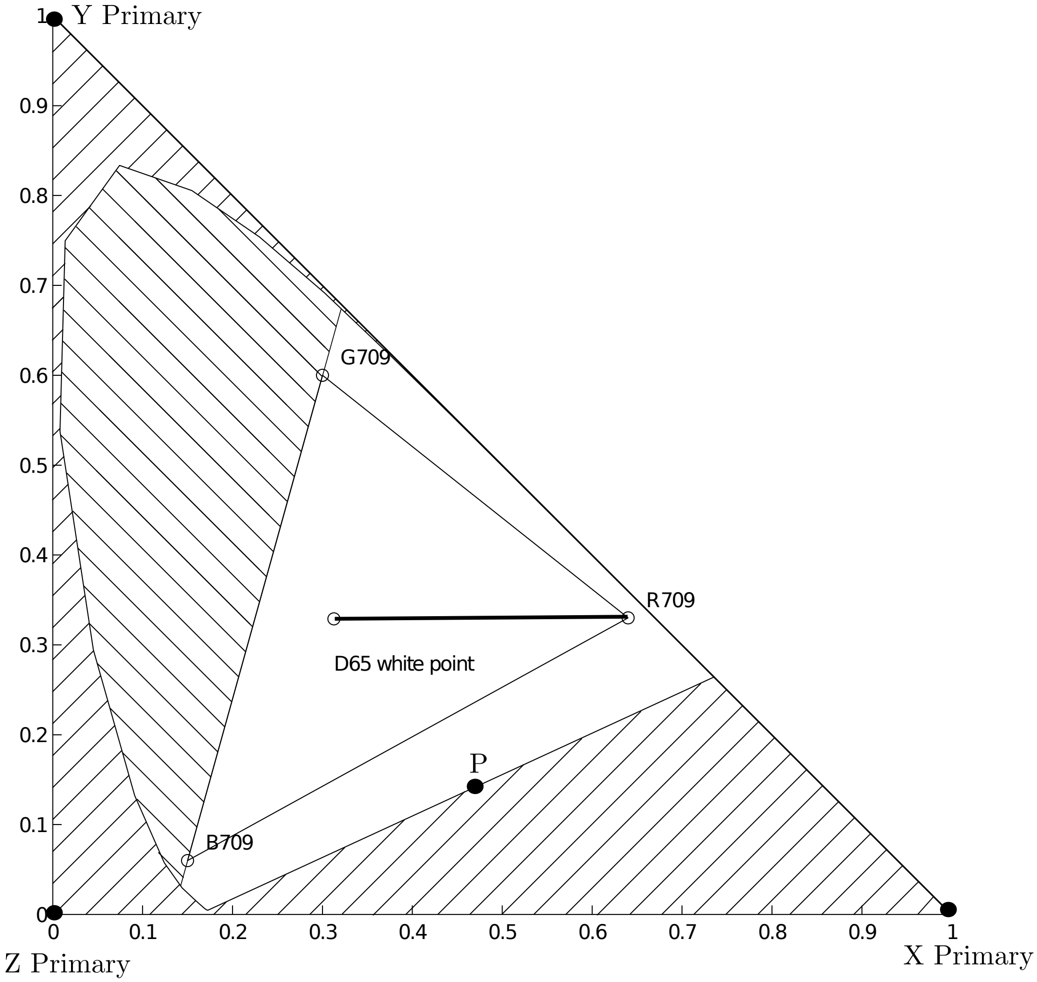
d) Imagine that an image with large energy in high frequencies is viewed from a great distance, and you would like to know the average color that the viewer sees. Should you low pass filter the L, a, b image to determine the average color? Either justify that this approach is correct, or explain a better approach.

Solution:

No, lowpass filtering (L, a, b) will produce the wrong results. The correct approach is to convert to (X, Y, Z), lowpass filter, and then convert back to (L, a, b).

Spring 2010 Final: Problem 1 (chromaticity diagram)

Consider the standard chromaticity diagram below, and assume that you are using a display device with standard 709 r, g, b color primaries. *Solution in figure below.*



a) Draw a triangle corresponding to all colors with positive values of X, Y, and Z. Label the three primaries for this triangle as "X-primary", "Y-primary", and "Z-primary".

b) Label the region of the chromaticity diagram corresponding to imaginary colors. (Use 45 deg diagonal hash marks to indicate this region of the diagram.)

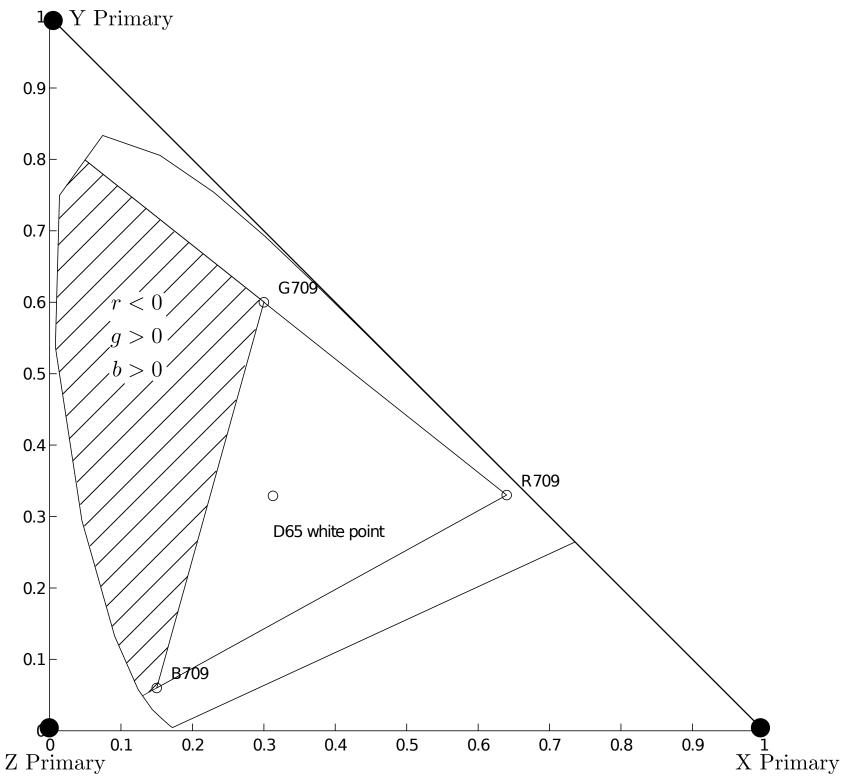
c) Label ALL real colors with $r < 0$ on the chromaticity diagram. (Use -45 deg diagonal hash marks to indicate this region of the diagram.)

d) Place a point on the diagram corresponding to a highly saturated color that is NOT formed by a single wavelength of light. Label this point with the letter "P".

e) Draw a line on the plot corresponding to all color that can be formed with a combination of the D65 white and R709.

Spring 2009 Exam 2: Problem 2 (chromaticity diagram)

Consider the standard chromaticity diagram below, and for all questions assume that standard 709 r, g, b color primaries are used. *Solution in figure below.*



a) Draw the region on the diagram corresponding to $r < 0$, $g > 0$, $b > 0$, and label this region "negative red".

b) Draw a point corresponding to the color primaries for X, Y, Z, and label the three points "X-primary", "Y-primary", and "Z-primary".

c) Draw a triangle corresponding to the gamut of the X, Y, Z color system, when the three tristimulus values are assumed positive.

d) Do all positive values of X, Y, Z correspond to real colors? Why or why not?

No. The spectral distribution of XYZ primaries are negative at some wavelengths, so the XYZ primaries are imaginary.

Not all positive values of X, Y, Z correspond to real colors.

e) Do all real colors correspond to positive values of X, Y, Z? Why or why not?

Yes, because all real colors are in the chromaticity diagram, which are inside the gamut of the XYZ color system.