Topics: Neighborhoods, connected components, clustering, and edge detection

Spring 2010 Exam 2: Problem 1 (edge detection)

Your objective is to perform edge detection on the sampled image $g(m,n) = f(mT, nT)$, where $f(x,y)$ is the associated continuous space image and $T = 1$. You will do this using a combination of gradient and Laplacian based operators.

a) Specify the condition for the detection of edges on the continuous image $f(x,y)$ using derivatives over x and y, and a single threshold γ.

Solution:

$$|\nabla f(x,y)| \geq \gamma \text{ and } \nabla^2 f(x,y) = 0$$

$$\left|\left|\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)\right|\right| \geq \gamma \qquad \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

b) Specify an approximate discretized gradient operator for the image $g(m,n)$.

Solution:

$$\frac{\partial f}{\partial x} \approx g(m+1,n) - g(m,n)$$
$$\frac{\partial f}{\partial y} \approx g(m,n+1) - g(m,n)$$

c) Specify an approximate discretized Laplacian operator for the image $g(m,n)$.

Solution:

$$\frac{\partial^2}{\partial x^2} \approx \frac{(g(m+1,n) - g(m,n)) - (g(m,n) - g(m-1,n))}{1}$$
$$= -2\left(g(m,n) - \frac{1}{2}\left(g(m+1,n) + g(m-1,n)\right)\right)$$
$$\nabla^2 f(x,y) \approx -4\left[g(m,n) - \frac{1}{4}\left(g(m+1,n) + g(m-1,n) + g(m,n+1) + g(m,n-1)\right)\right]$$

d) Specify the condition for the detection of edges on the discretized image $g(m,n)$ using approximate discretized gradient and Laplacian operators.

Solution:

$$(g(m+1,n) - g(m,n))^2 + (g(m,n+1) - g(m,n))^2 \geq \gamma^2$$

and

$$\text{Let } h(m,n) = g(m,n) - \frac{1}{4}\left(g(m+1,n) + g(m-1,n) + g(m,n+1) + g(m,n-1)\right)$$

$$\text{Then } h(m+1,n)h(m,n) < 0 \text{ or } h(m,n+1)h(m,n) < 0.$$

e) Describe how the threshold γ should be selected. What are the tradeoffs in its selection?

Solution:

As γ increases, probability of detection goes down and probability of false alarm goes down.

As γ decreases, probability of detection goes up and probability of false alarm goes up.

Spring 2007 Exam 2: Problem 3 (edge detection)

Consider the linear time-invariant discrete-time filter

$$y(n) = x(n) * h(n)$$

with input $x(n)$, output $y(n)$, and impulse response $h(n)$. Further, assume that $x(n)$ is created by sampling a continuous-time signal $s(t)$ as

$$x(n) = s(nT)$$

where $T = 1$.

a) Specify a simple FIR filter $h(n)$ so that $y(n)$ is approximately equal to $\left.\frac{ds(t)}{dt}\right|_{t=n-\frac{1}{2}}$.

Solution:

$$\left.\frac{ds(t)}{dt}\right|_{t=n-\frac{1}{2}} \approx \frac{s(n) - s(n-1)}{n - (n-1)} = s(n) - s(n-1)$$

$$\therefore y(n) = x(n) * h(n) = s(n) * h(n) = s(n) - s(n-1)$$

$$\Rightarrow h(n) = \delta(n) - \delta(n-1)$$

b) Specify a simple FIR filter $h(n)$ so that $y(n)$ is approximately equal to $\left.\frac{ds(t)}{dt}\right|_{t=n+\frac{1}{2}}$.

Solution:

$$\left.\frac{ds(t)}{dt}\right|_{t=n-\frac{1}{2}} \approx \frac{s(n+1) - s(n)}{(n+1) - n} = s(n+1) - s(n)$$

$$\therefore y(n) = x(n) * h(n) = s(n) * h(n) = s(n+1) - s(n)$$

$$\Rightarrow h(n) = \delta(n+1) - \delta(n)$$

c) Specify a simple FIR filter $h(n)$ so that $y(n)$ is approximately equal to $\left.\frac{d^2s(t)}{dt^2}\right|_{t=n}$.

Solution:

From a) and b), we have:

$$\left.\frac{d^2s(t)}{dt^2}\right|_{t=n} \approx \frac{\left.\frac{d(s)}{dt}\right|_{t=n+\frac{1}{2}} - \left.\frac{ds(t)}{dt}\right|_{t=n-\frac{1}{2}}}{\left(n+\frac{1}{2}\right) - \left(n-\frac{1}{2}\right)} \approx s(n+1) - s(n) - (s(n) - s(n-1))$$

$$= s(n+1) - 2s(n) + s(n-1)$$

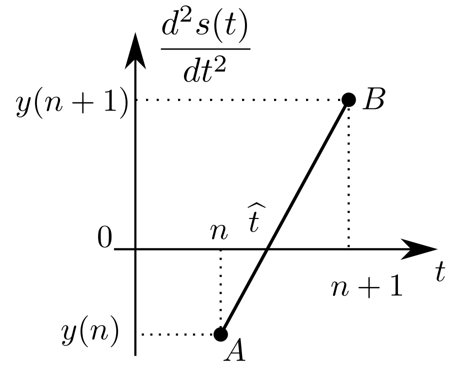
d) Specify an operation on $y(n)$ which determines when $\frac{d^2s(t)}{dt^2} = 0$ for some value of $n \leq t \leq n+1$.

Solution:

In c) we know that $y(n) \approx \left.\frac{d^2s(t)}{dt^2}\right|_{t=n}$, therefore

$$y(n+1) \approx \left.\frac{d^2s(t)}{dt^2}\right|_{t=n+1}$$

.



Suppose $s(t)$ is a smooth function on $[n, n+1]$.

If $y(n)y(n+1) < 0$, i.e., the sign changes, we can approximate $\frac{d^2s(t)}{dt^2}$ as the linear interpolation of the two points $(n, y(n))$, $(n+1, y(n+1))$ for $t \in [n, n+1]$.

$$\therefore \frac{\widehat{t} - n}{-y(n)} = \frac{n+1-\widehat{t}}{y(n+1)} \Rightarrow \widehat{t} = n + \frac{y(n)}{y(n) - y(n+1)}, \text{ s.t. } \left.\frac{d^2s(t)}{dt^2}\right|_{t=\widehat{t}} = 0$$

Spring 2004 Midterm Exam: Problem 2 (connected components)

Consider the following main program and subroutine.

Main Routine:

```
ClassLabel = 1
Initialize Y_r = 0 for r ∈ S
For each s ∈ S in raster order {
  if(Y_s = 0) {
    ConnectedSet(s, Y, ClassLabel)
    ClassLabel ← ClassLabel + 1
  }
}
```

Subroutine:

```
ConnectedSet(s_0, Y, ClassLabel) {
  B ← {s_0}
  While B is not empty {
    s ← any element of B
    B ← B − {s}
    Y_s ← ClassLabel
    B ← B ⋃ {r : r ∈ c(s) and Y_r = 0}
  }
  return(Y)
}
```

Also consider the following binary image

0	1	0	0	1
1	0	0	1	1
0	1	1	0	0
0	1	1	0	0
0	1	0	0	1

a) Calculate the output when the binary image is process by the main routine using a 4-pt neighborhood. Write your result in the table below.[1]

Solution:

1	2	3	3	4
5	3	3	4	4
6	3	7	8	8
6	7	7	8	8
6	7	8	8	9

b) Calculate the output when the binary image is process by the main routine using an 8-pt neighborhood. Write your result in the table below.[2]

Solution:

1	2	1	1	2
2	1	1	2	2
1	2	2	1	1
1	2	2	1	1
1	2	1	1	3

[1]Pixels on the image edge should be consider to have only 3 neighbors, and pixels in image corners should be considered to have only 2 neighbors.

[2]Pixels on the image edge should be consider to have only 5 neighbors, and pixels in image corners should be considered to have only 3 neighbors.