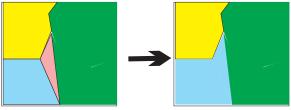
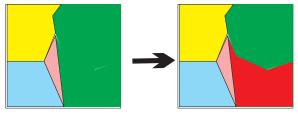
#### **Region Segmentation**

- Connected components analysis often results in many small disjointed regions.
- A connection (or break) at a single pixel can split (or merge) entire regions.
- There are three basic approaches to segmentation:
  - Region Merging recursively merge regions that are similar.



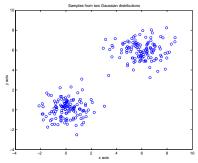
 Region Splitting - recursively divide regions that are heterogeneous.



 Split and merge - iteratively split and merge regions to form the "best" segmentation.

# **Hierarchical Clustering**

• Clustering refers to techniques for separating data samples into sets with distinct characteristics.



- Clustering methods are analogous to segmentation methods.
  - Agglomerative clustering "bottom up" procedure for recursively merging clusters ⇒ analogous to region merging
  - Divisive clustering "top down" procedure for recursively splitting clusters ⇒ analogous to region splitting

#### **Image Regions and Partitions**

- Let  $R_m \subset S$  denote a region of the image where  $m \in \mathcal{M}$ .
- We say that  $\{R_m|m\in\mathcal{M}\}$  partitions the image if

For all 
$$m \neq k$$
,  $R_m \cap R_k = \emptyset$ 

$$\bigcup_{m \in \mathcal{M}} R_m = S$$

ullet Each region  $R_m$  has **features** that characterize it.

#### **Typical Region Features**

#### Color

- Mean RGB value
- 1-D color histograms in R, G, and B
- 3-D color histogram in (R,G,B)

#### Texture

- Spatial autocorrelation
- Joint probability distribution for neighboring pixels (e.g. the spatial co-occurrence matrix)
- Wavelet transform coefficients

#### Shape

- Number of pixels
- Width and height attributes
- Boundary smoothness attributes
- Adjacent region labels

#### **Recursive Feature Computation**

• Any two regions may be merged into a new region.

$$R_{new} = R_k \cup R_l$$

- Let  $f_n = f(R_n) \in \mathbb{R}^k$  be a k dimensional feature vector extracted from the region  $R_n$ .
- Ideally, the features of merged regions may be computed without reference to the original pixels in the region.

$$f(R_k \cup R_l) = f(R_k) \oplus f(R_l)$$
$$f_{new} = f_k \oplus f_l$$

here  $\oplus$  denotes some operation on the values of the two feature vectors.

### **Example of Recursive Feature Computation**

Example: Let  $f(R_k) = (N_k, \mu_k, c_k)$  where

$$N_k = |R_k|$$

$$\mu_k = \frac{1}{N_k} \sum_{s \in R_k} x_s$$

$$c_k = \frac{1}{N_k} \sum_{s \in R_k} s$$

We may compute the region features for  $R_{new} = R_k \cup R_l$  using the recursions

$$N_{new} = N_k + N_l$$

$$\mu_{new} = \frac{N_k \mu_k + N_l \mu_l}{N_{new}}$$

$$c_{new} = \frac{N_k c_k + N_l c_l}{N_{new}}$$

# **Recursive Merging**

• Define a distance function between regions. In general, this function has the form

$$d_{k,l} = D(R_k, R_l) \ge 0$$

• Ideally,  $D(R_k, R_l)$  is **only** a function of the feature vectors  $f_k$  and  $f_l$ .

$$d_{k,l} = D(f_k, f_l) \ge 0$$

• Then merge regions with minimum distance.

#### **Example of Merging Criteria**

• Distance between color means

$$d_{k,l} = \frac{N_k}{N_{new}} |\mu_k - \mu_{new}|^2 + \frac{N_l}{N_{new}} |\mu_l - \mu_{new}|^2$$

• Distance between region centers

$$d_{k,l} = \frac{N_k}{N_{new}} |c_k - c_{new}|^2 + \frac{N_l}{N_{new}} |c_l - c_{new}|^2$$

• Distance formed by a weighted combination of the two

$$d_{k,l} = \alpha \left( \frac{N_k}{N_{new}} |\mu_k - \mu_{new}|^2 + \frac{N_l}{N_{new}} |\mu_l - \mu_{new}|^2 \right) + \beta \left( \frac{N_k}{N_{new}} |c_k - c_{new}|^2 + \frac{N_l}{N_{new}} |c_l - c_{new}|^2 \right)$$

#### **Recursive Merging Algorithm**

• Define a distance function between regions

$$d_{k,l} = D(f(R_k), f(R_l)) > 0$$

Repeat until  $|\mathcal{M}| = 1$  {

Determine the minimum distance regions

$$(k^*, l^*) = \arg\min_{k,l \in \mathcal{M}} \{d_{k,l}\}$$

Merge the minimum distance regions

$$R_{k^*} \leftarrow R_{k^*} \cup R_{l^*}$$

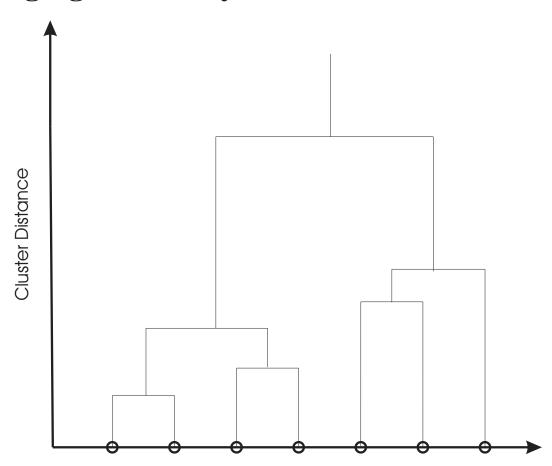
Remove unused region

$$\mathcal{M} \leftarrow \mathcal{M} - \{l^*\}$$

}

• This recursion generates a binary tree.

#### **Merging Hierarchy and Order Identification**



• Clustering can be terminated when the distance exceeds a threshold

$$d_{k^*,l^*} > Threshold \Rightarrow Stop clustering$$

• Different thresholds result in different numbers of clusters.