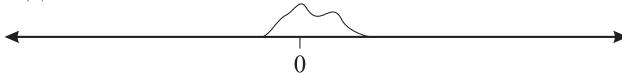
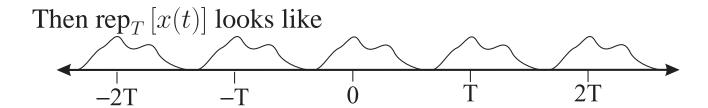
1-D Rep Operation

The rep operator periodically replicates a function with some specified period T.

$$\operatorname{rep}_T\left[x(t)\right] = \sum_{k=-\infty}^{\infty} x(t-kT)$$

If x(t) looks like





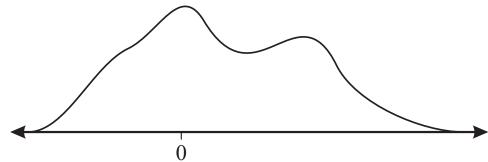
The resulting function is periodic with period T.

1-D Comb Operation

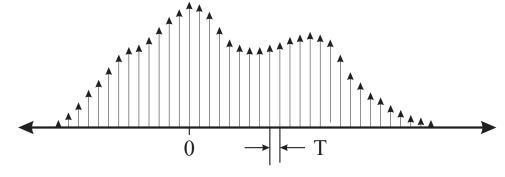
The *comb* operator multiplies a function by a periodic train of impulses.

$$\begin{aligned} \operatorname{comb}_{T}\left[x(t)\right] &= \sum_{k=-\infty}^{\infty} \delta(t-kT)x(t) \\ &= x(t) \sum_{k=-\infty}^{\infty} \delta(t-kT) \end{aligned}$$





Then $comb_T [x(t)]$ looks like



The spacing between impulses is T.

1-D Rep and Comb Transform Properties

Assume that:

$$x(t) \stackrel{CTFT}{\Leftrightarrow} X(f)$$

Then the transform relationship is:

$$\operatorname{comb}_{T}\left[x(t)\right] \ \stackrel{CTFT}{\Leftrightarrow} \ \frac{1}{T}\operatorname{rep}_{\frac{1}{T}}\left[X(f)\right]$$

$$\operatorname{rep}_{T}\left[x(t)\right] \ \stackrel{CTFT}{\Leftrightarrow} \ \frac{1}{T} \mathrm{comb}_{\frac{1}{T}}\left[X(f)\right]$$

2-D Rep and Comb Operators

2-D Rep function:

$$\operatorname{rep}_{X,Y}\left[f(x,y)\right]$$

$$=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}f(x-mX,y-nY)$$

2-D Comb function:

$$comb_{X,Y} [f(x,y)]$$

$$= f(x,y) \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x - mX, y - nY)$$

2-D Rep and Comb Transform Properties

Assume that:

$$f(x,y) \stackrel{CSFT}{\Leftrightarrow} F(u,v)$$

Then the transform relationship is:

$$\begin{aligned} & \operatorname{comb}_{X,Y}\left[f(x,y)\right] & \overset{CSFT}{\Leftrightarrow} & \frac{1}{XY} \mathrm{rep}_{\frac{1}{X},\frac{1}{Y}}\left[F(u,v)\right] \\ & \operatorname{rep}_{X,Y}\left[f(x,y)\right] & \overset{CSFT}{\Leftrightarrow} & \frac{1}{XY} \mathrm{comb}_{\frac{1}{X},\frac{1}{Y}}\left[F(u,v)\right] \end{aligned}$$