

# Magnetic Resonance Imaging (MRI)



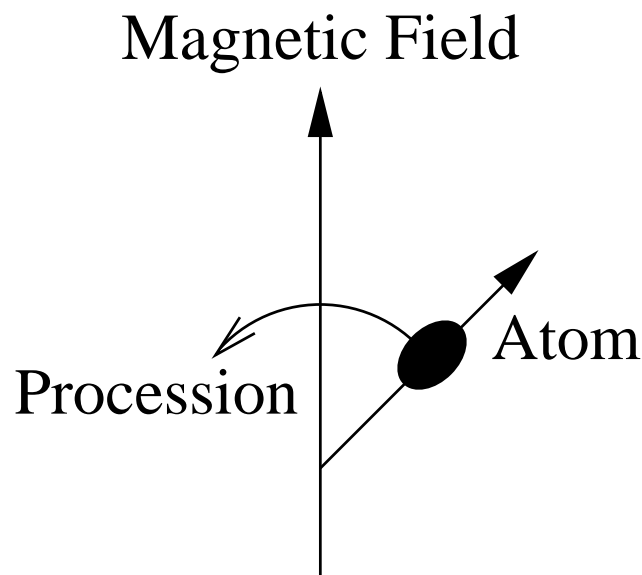
- Can be very high resolution
- No radiation exposure
- Very flexible and programmable
- Tends to be expensive, noisy, slow



## MRI Attributes

- Based on magnetic resonance effect in atomic species
- Does not require any ionizing radiation
- Numerous modalities
  - Conventional anatomical scans
  - Functional MRI (fMRI)
  - MRI Tagging
- Image formation
  - RF excitation of magnetic resonance modes
  - Magnetic field gradients modulate resonance frequency
  - Reconstruction computed with inverse Fourier transform
  - Fully programmable
  - Requires an enormous (and very expensive) superconducting magnet

## Magnetic Resonance



- Atom will precess at the Lamor frequency

$$\omega_o = LM$$

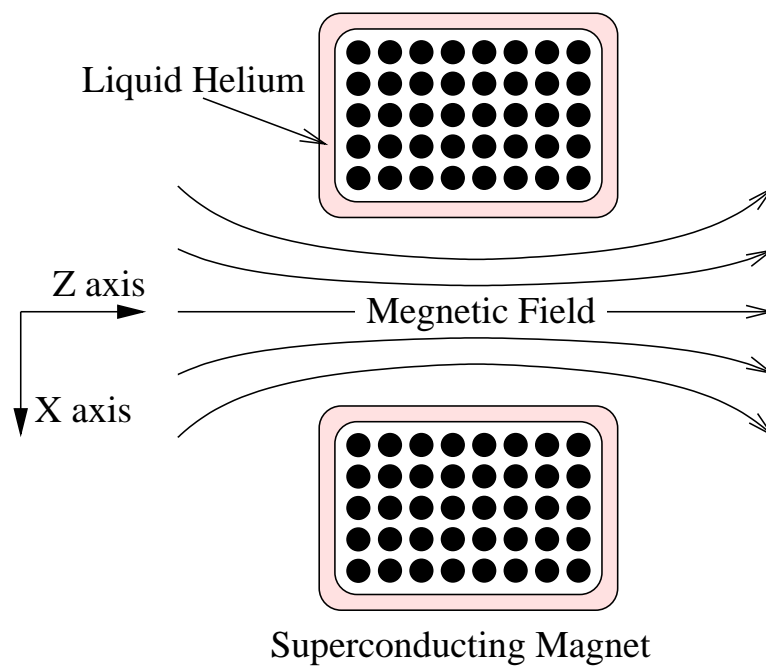
- Quantities of importance

$M$  - magnitude of ambient magnetic field

$\omega_o$  - frequency of precession (radians per second)

$L$  - Lamor constant. Depends on choice of atom

## The MRI Magnet



- Large super-conducting magnet
  - Uniform field within bore
  - Very large static magnetic field of  $M_o$

## Magnetic Field Gradients

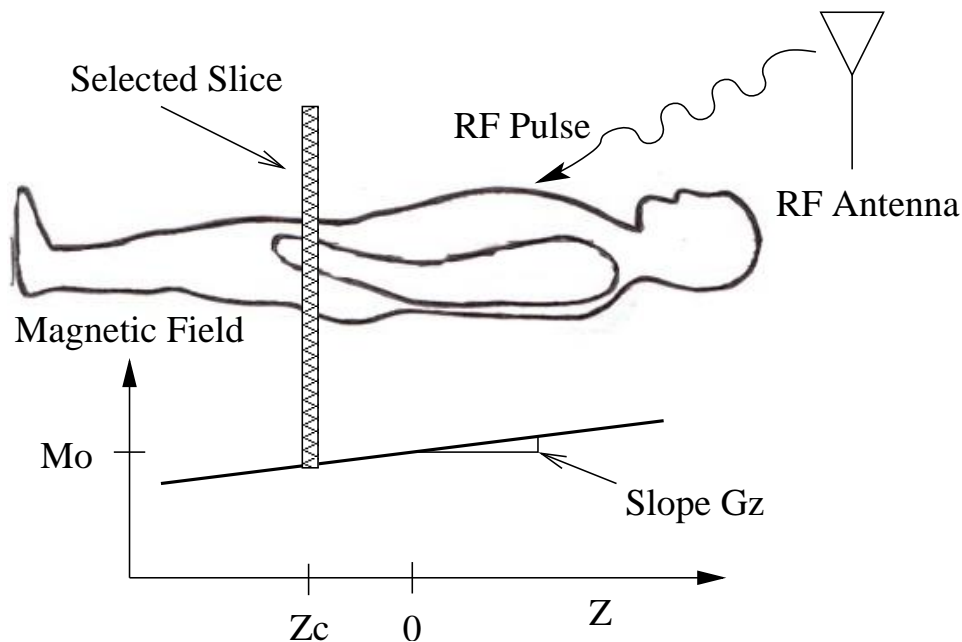
- Magnetic field **magnitude** at the location  $(x, y, z)$  has the form

$$M(x, y, z) = M_o + xG_x + yG_y + zG_z$$

- $G_x$ ,  $G_y$ , and  $G_z$  control magnetic field gradients
  - Gradients can be changed with time
  - Gradients are small compared to  $M_o$
- For time varying gradients

$$M(x, y, z, t) = M_o + xG_x(t) + yG_y(t) + zG_z(t)$$

## MRI Slice Select



- Design RF pulse to excite protons in single slice
  - Turn off  $x$  and  $y$  gradients, i.e.  $G_x = G_y = 0$ .
  - Set  $z$  gradient to fix positive value,  $G_z > 0$ .
  - Use the fact that resonance frequency is given by

$$\omega = L (M_o + zG_z) .$$

## Slice Select Pulse Design

- Design parameters
  - Slice center =  $z_c$ .
  - Slice thickness =  $\Delta z$ .
- Slice centered at  $z_c \Rightarrow$  pulse center frequency

$$f_c = \frac{LM_o}{2\pi} + \frac{z_c LG_z}{2\pi} = f_o + \frac{z_c LG_z}{2\pi} .$$

- Slice thickness  $\Delta z \Rightarrow$  pulse bandwidth

$$\Delta f = \frac{\Delta z LG_z}{2\pi} .$$

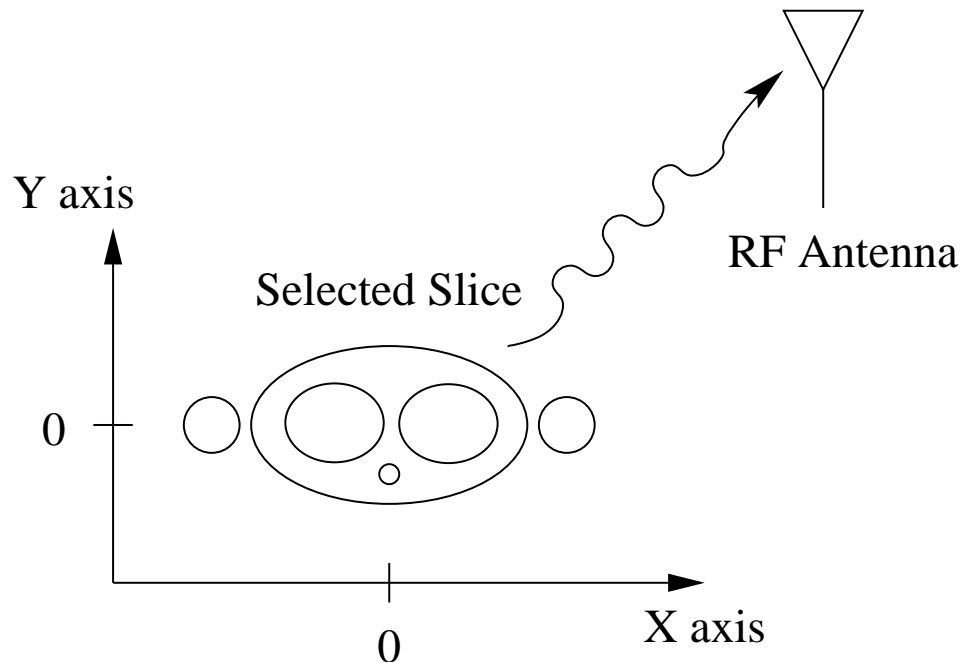
- Using these parameters, the pulse is given by

$$s(t) = e^{j2\pi f_c t} \text{sinc}(t\Delta f)$$

and its CTFT is given by

$$S(f) = \text{rect}\left(\frac{(f - f_c)}{\Delta f}\right)$$

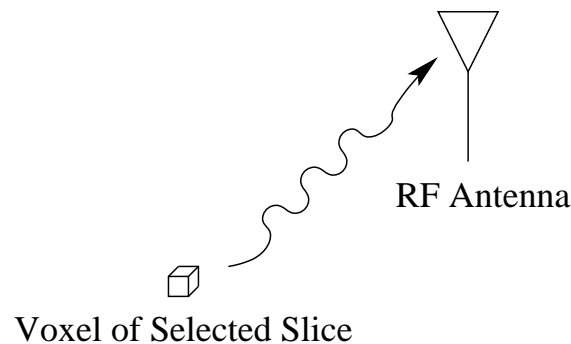
## How Do We Imaging Selected Slice?



- Precessing atoms radiate electromagnetic energy at RF frequencies
- Strategy
  - Vary magnetic gradients along  $x$  and  $y$  axes
  - Measure received RF signal
  - Reconstruct image from RF measurements



## Signal from a Single Voxel



- RF signal from a single voxel has the form

$$r(x, y, t) = f(x, y)e^{j\phi(t)}$$

$f(x, y)$  voxel dependent weighting

- Depends on properties of material in voxel
- Quantity of interest
- Typically “weighted” by T1, T2, or T2\*

$\phi(t)$  phase of received signal

- Can be modulated using  $G_x$  and  $G_y$  magnetic field gradients
- We assume that  $\phi(0) = 0$

## Analysis of Phase

- Frequency = time derivative of phase

$$\frac{d\phi(t)}{dt} = L M(x, y, t)$$

$$\phi(t) = \int_0^t L M(x, y, \tau) d\tau$$

$$= \int_0^t L M_o + x L G_x(\tau) + y L G_y(\tau) d\tau$$

$$= \omega_o t + x k_x(t) + y k_y(t)$$

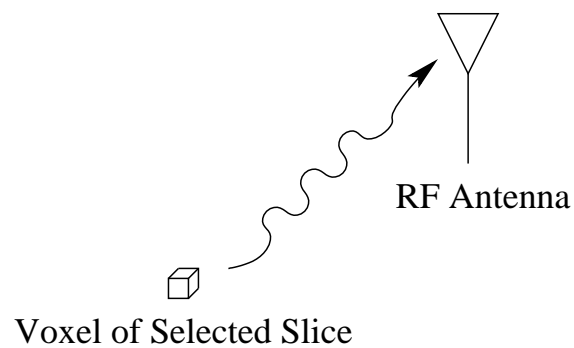
where we define

$$\omega_o = L M_o$$

$$k_x(t) = \int_0^t L G_x(\tau) d\tau$$

$$k_y(t) = \int_0^t L G_y(\tau) d\tau$$

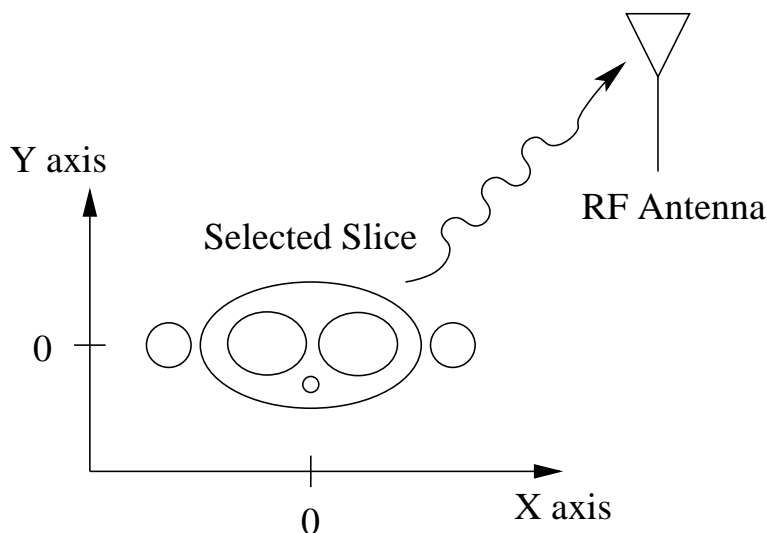
## Received Signal from Voxel



- RF signal from a single voxel has the form

$$\begin{aligned} r(t) &= f(x, y)e^{j\phi(t)} \\ &= f(x, y)e^{j(\omega_0 t + xk_x(t) + yk_y(t))} \\ &= f(x, y)e^{j\omega_0 t} e^{j(xk_x(t) + yk_y(t))} \end{aligned}$$

## Received Signal from Selected Slice



- RF signal from the complete slice is given by

$$\begin{aligned}
 R(t) &= \int_{\mathbb{R}} \int_{\mathbb{R}} r(x, y, t) dx dy \\
 &= \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j\omega_0 t} e^{j(xk_x(t) + yk_y(t))} dx dy \\
 &= e^{j\omega_0 t} \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j(xk_x(t) + yk_y(t))} dx dy \\
 &= e^{j\omega_0 t} F(-k_x(t), -k_y(t))
 \end{aligned}$$

where  $F(u, v)$  is the CSFT of  $f(x, y)$

## K-Space Interpretation of Demodulated Signal

- RF signal from the complete slice is given by

$$F(-k_x(t), -k_y(t)) = R(t)e^{-j\omega_0 t}$$

where

$$k_x(t) = \int_0^t LG_x(\tau) d\tau$$

$$k_y(t) = \int_0^t LG_y(\tau) d\tau$$

- Strategy
  - Scan spatial frequencies by varying  $k_x(t)$  and  $k_y(t)$
  - Reconstruct image by performing (inverse) CSFT
  - $G_x(t)$  and  $G_y(t)$  control velocity through K-space

## Controlling K-Space Trajectory

- Relationship between gradient coil voltage and K-space

$$L_x \frac{di(t)}{dt} = v_x(t) \quad G_x(t) = M_x i(t)$$

$$L_y \frac{di(t)}{dt} = v_y(t) \quad G_y(t) = M_y i(t)$$

using this results in

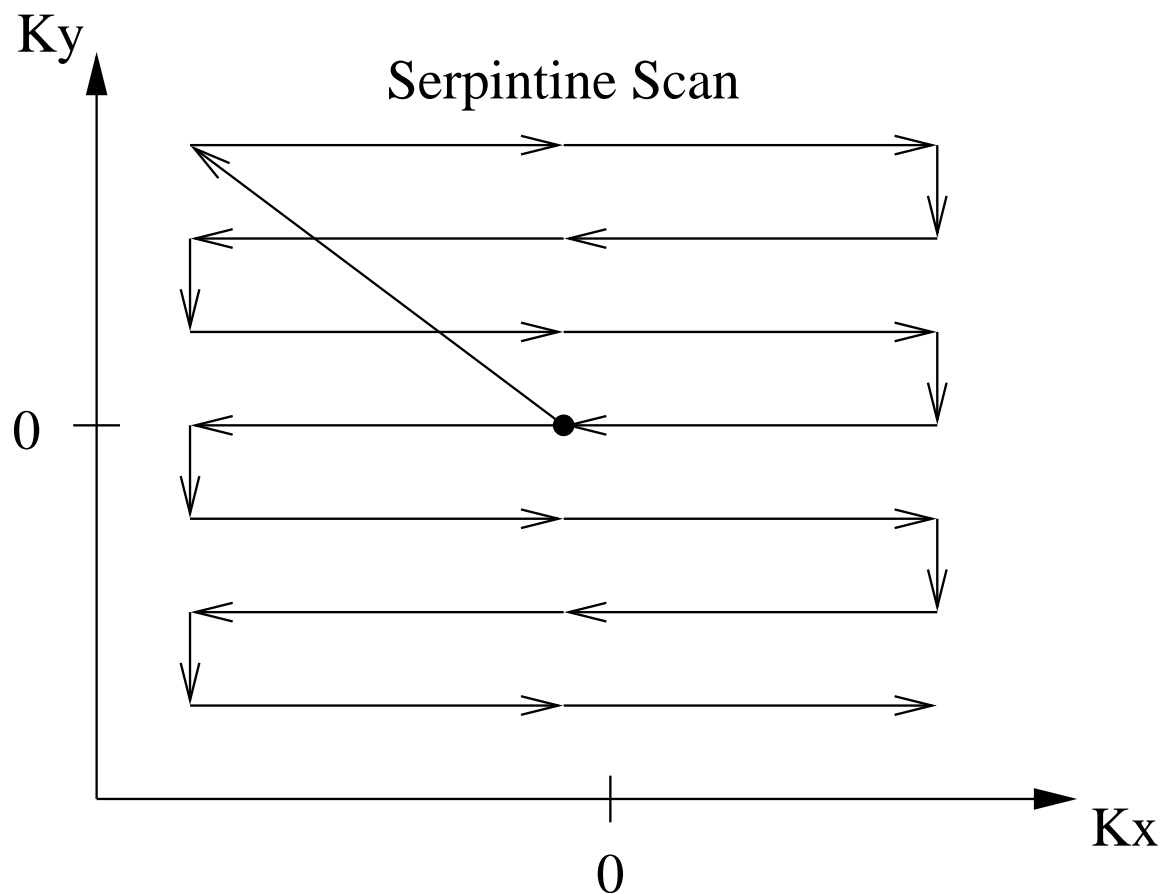
$$k_x(t) = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau$$

$$k_y(t) = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau$$

- $v_x(t)$  and  $v_y(t)$  are like the accelerator peddles for  $k_x(t)$  and  $k_y(t)$

## Echo Planer Imaging (EPI) Scan Pattern

- A commonly used raster scan pattern through K-space

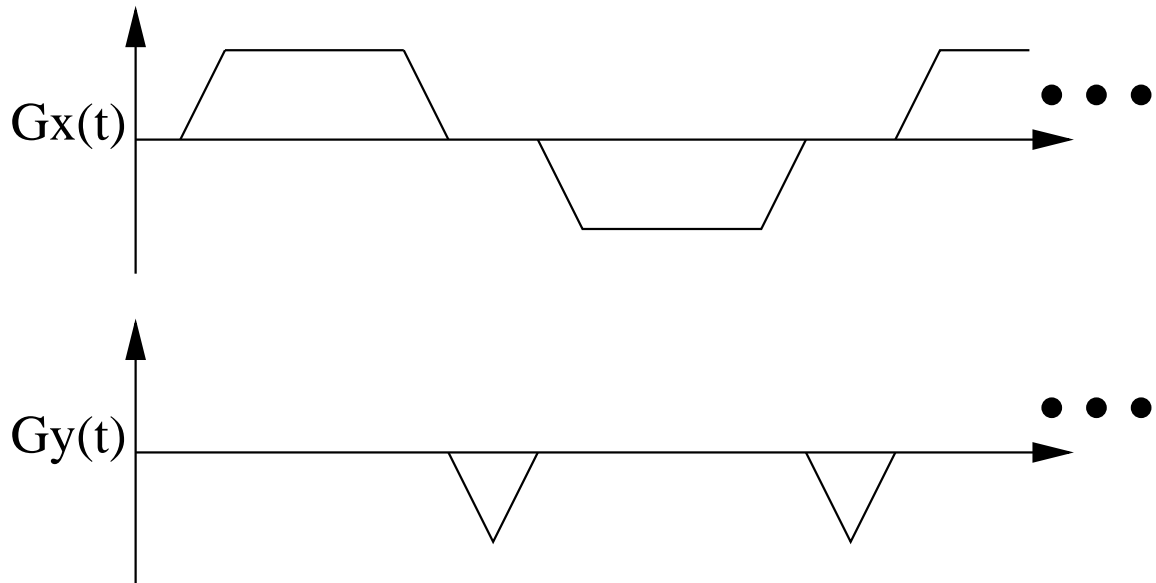


$$k_x(t) = L \int_0^t G_x(\tau) d\tau = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau$$

$$k_y(t) = L \int_0^t G_y(\tau) d\tau = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau$$

## Gradient Waveforms for EPI

- Gradient waveforms in  $x$  and  $y$  look like



- Voltage waveforms in  $x$  and  $y$  look like

