Topics: Achromatic Vision, Gamma, and Visual MTF

Spring 2009 Exam 2: Problem 3 (gamma correction)

Let $T[\cdot]$ be the gamma correction function for $\gamma = 2.2$, and let $T^{-1}[\cdot]$ be its inverse. Furthermore, let $X(m,n)$ be a gray scale image which is linear in energy scaled to the $[0,1]$ range; assume that $T[1] = 1$ and $T[0] = 0$; and also assume that $X(m,n) = 1$ corresponds to white, and $X(m,n) = 0$ corresponds to black.

Then the gamma corrected version of the image is given by

$$\widetilde{X}(m,n) = T[X(m,n)]$$

From this data, two different images are formed.

$$\begin{aligned} \widetilde{Y}_1(m,n) &= h(m,n) * \widetilde{X}(m,n) \\ Y_2(m,n) &= h(m,n) * X(m,n) \end{aligned}$$

where $*$ denotes 2D convolution, and $h(m,n)$ is a low pass filter with an approximate cut-off at $\sqrt{\mu^2 + \nu^2} = \pi/100$. The result, $Y_2(m,n)$, is then gamma corrected to form

$$\widetilde{Y}_2(m,n) = T[Y_2(m,n)] \ .$$

For all problems, assume that all displays have a gamma of $\gamma = 2.2$.

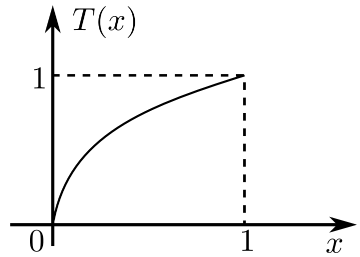
a) Assuming that we use a conventional FIR filter implementation, approximately how many multiplies per pixel will it take to implement this filter?

Solution:

If we have a 1D lowpass filter of length N, then the cutoff frequency will be approximately $\frac{2\pi}{N}$. So in the 2D case, a lowpass filter of size $200x200$ will give us an approximate cutoff frequency of $\frac{2\pi}{200} = \frac{\pi}{100}$. Therefore, approximately $200x200 = 40,000$ multiplies will be needed for each output pixel. This calculation is for a conventional FIR filter, not a separable one.

b) Sketch a plot of the gamma correction function $y = T[x]$, for x in the range of $[0, 1]$.

Solution:



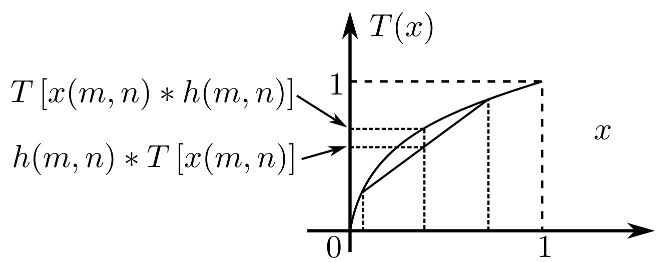
c) In general, which image is brighter when displayed, $\widetilde{Y}_1(m,n)$ or $\widetilde{Y}_2(m,n)$? Why?

Solution:

In general , $\widetilde{Y_2}(m,n)$ is brighter.

Reason: $T[\cdot]$ is a concave function. Convolution with a low-pass filter is a linear operation, so

$$T[h(m,n) * x(m,n)] \geq h(m,n) * T[x(m,n)]$$



$$\widetilde{Y_1}(m,n) = h(m,n) * T[X(m,n)]$$

$$\widetilde{Y_2}(m,n) = T[h(m,n) * X(m,n)]$$

So $\widetilde{Y_2}(m,n) \geq \widetilde{Y_1}(m,n)$

d) Assuming that $h(m,n)$ is used to represent the MTF of the human visual system, which of the two images, $\widetilde{Y}_1(m,n)$ or $\widetilde{Y}_2(m,n)$, would you expect to more accurately match the original image $\widetilde{X}(m,n)$ when displayed on a calibrated monitor?

Solution:

$\widetilde{Y_2}(m,n)$ will more accurately the original image $\widetilde{X}(m,n)$ when displayed on a calibrated monitor.

e) Justify your answer to part d).

Solution:

Photon energy adds up linearly, so the human eye can be modeled as a low-pass filter in the linear space, not the gamma corrected space.

When displayed on the monitor, $\widetilde{Y_2}(m,n)$ becomes

$$T^{-1}\left[\widetilde{Y_2}(m,n)\right] = T^{-1}\left[T\left[h(m,n) * X(m,n)\right]\right] = h(m,n) * X(m,n)$$

When displayed on the monitor, $\widetilde{X}(m,n)$ becomes

$$T^{-1}\left[\widetilde{X}(m,n)\right] = T^{-1}\left[T\left[X(m,n)\right]\right] = X(m,n)$$

Then considering the low-pass feature of human vision, it is perceived as $h(m,n) * X(m,n)$.

Therefore, $\widetilde{Y_2}(m,n)$ is a better match. Note that $\widetilde{Y_1}$(m,n) does low-pass filtering in gamma corrected space, so it is not a good match.

Spring 2007 Exam 2: Problem 2 (contrast)

Consider an image display system where Y represents the luminance of the output light in units proportional to energy.

When the background luminance is $Y = 10$ a viewer can just notice a spot when it has a luminance of $Y_s = 10.1$, and it has a diameter of 10 degrees in units of visual subtended angle.

a) What is the just noticeable contrast?

Solution:

$$\Delta Y_{JND} = Y_S - Y = 0.1 \Rightarrow C_{JND} = \frac{\Delta Y_{JND}}{Y} = \frac{0.1}{10} = 0.01$$

b) What is the contrast sensitivity?

Solution:

$$\text{Contrast sensitivity } S = \frac{1}{C_{JND}} = 100$$

c) Assuming that Weber's Law holds true, what luminance must the spot have in order to be just noticeable when the background luminance is $Y = 1$?

Solution:

Weber's Law says the contrast sensitivity S is approximately independent of the background luminance.

$$\therefore S = \frac{Y}{\Delta Y_{JND}} = \text{constant} = 100$$

$$\therefore \text{When } Y = 1,\ \Delta Y_{JND} = 0.01 \Rightarrow Y_S = 1.01$$

d) Select a function $f(Y)$ so that equal quantization steps in the value of $f(Y)$ represent equal changes in contrast. Justify your selection.

Solution:

$$\Delta\left(f(Y)\right) = \frac{\Delta Y}{Y}$$

$$d\left(f(Y)\right) = \frac{1}{Y} dY$$

$$\int d\left(f(Y)\right) = \int \frac{1}{Y} dY$$

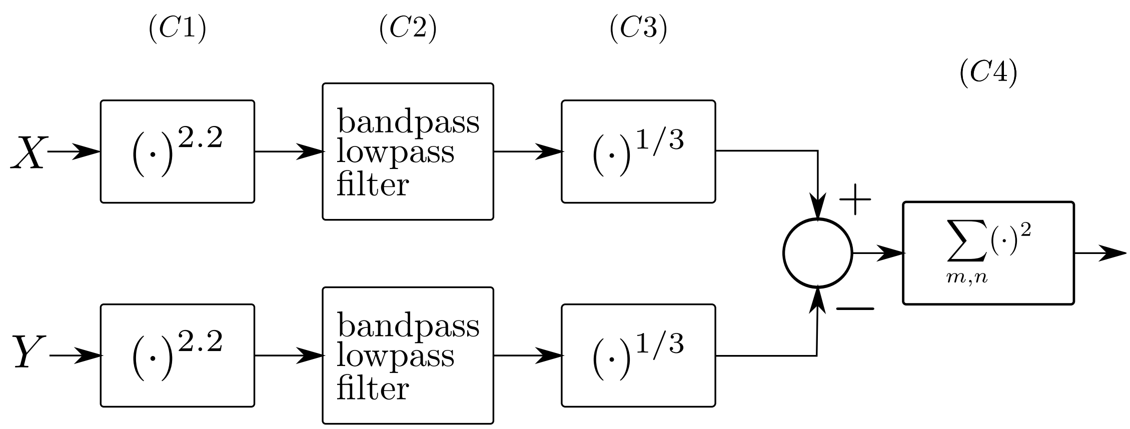
$$\Rightarrow f(Y) = \log(Y)$$

Spring 2004 Midterm Exam: Problem 4 (MTF and gamma correction)

Specify a system based on a simple image fidelity model for achromatic images. The systems should:

- Have two inputs consisting of two γ-corrected images, with $\gamma = 2.2$.
- Account for the MTF of the human visual system.
- Account for perceptual sensitivity to contrast.
- Have a single scalar output.

a) Give a block diagram for this system, and specify each block's operation.



Solution:

b) Explain why each major component is required. When appropriate, give examples of what would go wrong if a component was not used.

Solution:

(C1) This component converts to linear coordinates. Without this component, the linear filter would cause tone shifts.

(C2) This component removes high spatial frequencies that are not visible.

C3) This component adjusts for the visual system's sensitivity to contrast. Without this component, dark regions would be under-represented.

C4) This component integrates together the squared error.

c) Give examples of an application where this system might be useful.

Solution:

Two examples are image coding and halftoning. In both cases, it is often necessary to determine the visual difference between an original and processed image.