EE 637 Midterm II

April 9, Spring 2010

Name: ____________________________________

Instructions:

- This is a 50 minute exam containing **three** problems.
- You may **only** use your brain and a pencil (or pen) to complete this exam.
- You **may not** use your book, notes, or a calculator.

Good Luck.

Fact Sheet

- Function definitions

$$\mathrm{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1-|t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFT

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \\ x(t) &= \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \end{aligned}$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X(-f)$$

$$x(t-t_0) \overset{CTFT}{\Leftrightarrow} X(f) e^{-j2\pi f t_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|} X(f/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} x(-f)$$

$$x(t) e^{j2\pi f_0 t} \overset{CTFT}{\Leftrightarrow} X(f-f_0)$$

$$x(t) y(t) \overset{CTFT}{\Leftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(f) Y(f)$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) Y^*(f) df$$

- CTFT pairs

$$\mathrm{sinc}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{rect}(f)$$

$$\mathrm{rect}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{sinc}(f)$$

For $a > 0$

$$\frac{1}{(n-1)!} t^{n-1} e^{-at} u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{(j2\pi f + a)^n}$$

- CSFT

$$\begin{aligned} F(u,v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx dy \\ f(x,y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} du dv \end{aligned}$$

- DTFT

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \end{aligned}$$

- DTFT pairs

$$a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{1 - a e^{-j\omega}}$$

- Rep and Comb relations

$$\mathrm{rep}_T\left[x(t)\right] = \sum_{k=-\infty}^{\infty} x(t-kT)$$

$$\mathrm{comb}_T\left[x(t)\right] = x(t) \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$\mathrm{comb}_T\left[x(t)\right] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \mathrm{rep}_{\frac{1}{T}}\left[X(f)\right]$$

$$\mathrm{rep}_T\left[x(t)\right] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \mathrm{comb}_{\frac{1}{T}}\left[X(f)\right]$$

- Sampling and Reconstruction

$$\begin{aligned} y(n) &= x(nT) \\ Y(e^{j\omega}) &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right) \\ s(t) &= \sum_{k=-\infty}^{\infty} y(k) \delta(t-kT) \\ S(e^{j\omega}) &= Y\left(e^{j\omega T}\right) \end{aligned}$$

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Problem 1.(33pt)

Your objective is to perform edge detection on the sampled image $g(m,n) = f(mT, nT)$, where $f(x,y)$ is the associated continuous space image and $T = 1$. You will do this using a combination of gradient and Laplacian based operators.

a) Specify the condition for the detection of edges on the continuous image $f(x,y)$ using derivatives over x and y, and a single threshold γ.
b) Specify an approximate discretized gradient operator for the image $g(m,n)$.
c) Specify an approximate discretized Laplacian operator for the image $g(m,n)$.
d) Specify the condition for the detection of edges on the discretized image $g(m,n)$ using approximate discretized gradient and Laplacian operators.
e) Describe how the threshold γ should be selected. What are the tradeoffs in its selection?

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Problem 2.(34pt)

Consider the color display that produces the following color (X, Y, Z) when given the input is (r, g, b).

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (r/255)^\alpha \\ (g/255)^\alpha \\ (b/255)^\alpha \end{bmatrix}$$

a) What is the gamma of the display?
b) What is the white point of the display?
c) What are the color primaries of the display?
d) Can such a display be physically built? Why or why not?
e) You produce two images, one with a checker board pattern of values 0 and 255, and a second with a constant value of $(r, g, b) = (a, a, a)$. The value of a is then adjusted so that the two images match when viewed from a distance. What is the value of gamma for the display in terms of the value a?
f) Imagine that $\alpha = 1$ and the values of (r, g, b) are quantized to 8-bits. Describe the defects you would expect to see in the displayed image.

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Problem 3.(33pt) Consider a sampling system were the input, $s(t) = \text{sinc}(2t)$, is sampled with period $T = 1/2$ to form the sampled signal $x(n) = s(nT)$.

After sampling, you determine that you selected the wrong sampling rate, and really need to have sampled the signal at the period $T_2 = 1/4$; so you interpolate by a factor of $L = 2$ to form the signal $y(n)$.

a) Sketch the signal $s(t)$ and its CTFT $S(f)$. What is the Nyquist sampling rate for this signal?

b) Sketch the signal $x(n)$ and also sketch its DTFT $X(e^{j\omega})$.

c) Sketch $x(n)$ after it is up-sampled by $L = 2$.

d) Sketch the interpolation filter's impulse response.

e) Sketch the signal $y(n)$.

f) What is the relationship between $y(n)$ and $s(t)$?