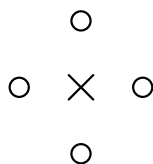


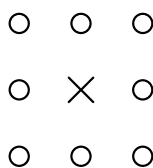
## 2-D Neighborhoods

- 4-point neighborhood



$$\partial(i, j) = \{(i + 1, j), (i - 1, j), (i, j + 1), (i, j - 1)\}$$

- 8-point neighborhood



$$\partial(i, j) = \left\{ \begin{array}{l} (i + 1, j), (i - 1, j), (i, j + 1), (i, j - 1) \\ (i + 1, j + 1), (i - 1, j + 1) \\ (i + 1, j - 1), (i - 1, j - 1) \end{array} \right\}$$

- More generally, a *Neighborhood System* is any mapping with the two properties that:

1. For all  $s \in S$ ,  $s \notin \partial s$
2. For all  $r \in S$ ,  $r \in \partial s \Rightarrow s \in \partial r$

## Boundary Conditions

- How do you process pixels on the boundary of an image??
- Consider the following example using a 4-point neighborhood

A small example image

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>

4 neighbors of *l*

	<i>l</i> <sub>1</sub>	
<i>l</i> <sub>4</sub>	<i>l</i>	<i>l</i> <sub>2</sub>
	<i>l</i> <sub>3</sub>	

- Free boundary condition

$$\partial l = \{h, p, k\}$$

$$\partial p = \{l, o\}$$

- Toriodal boundary condition (asteroids)

$$\partial l = \{h, i, p, k\}$$

$$\partial p = \{l, m, d, o\}$$

- Reflective boundary condition

$$l_1 = h, l_2 = k, l_3 = p, l_4 = k$$

$$p_1 = l, p_2 = o, p_3 = l, p_4 = o$$

## Edge Detection

- Edges
  - Edges naturally occur in images due to the discontinuities form by occlusion.
  - Edges often delineate the boundaries between distinct regions.
  - Edges often contain important visual and semantic information.
- Edge detection:
  - The process of identifying pixels that fall along edges.
  - As with any detect process subject to a trade-off between false alarm and missed detection rates.
- Performance Metrics:
  - Evaluation of edge detection schemes can be difficult.
  - Correct labeling of edge and non-edge pixels often requires subjective interpretation.
  - Best choice of edge detection scheme usually depends on task.
  - Performance metrics exist and usually use synthetic data input for evaluation.

## Gradient Based Edge Detection

- Compute local estimate of gradient

$$\nabla f(x, y) = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

- From these, compute gradient magnitude and angle.

$$|\nabla f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

$$\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

- Apply threshold to the magnitude of gradient

$$\text{edge} \quad |\nabla f| \geq T$$

$$\text{no edge} \quad |\nabla f| < T$$

- Choosing  $T$ 
  - Too large  $\Rightarrow$  missed detections
  - Too small  $\Rightarrow$  false alarms

## How to Compute Gradient

- Directional derivatives can be computed by applying a spatial filter.
- Conventional (off center)

$$\begin{bmatrix} \boxed{-1} & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} \boxed{-1} & 0 \\ 1 & 0 \end{bmatrix}$$

- Roberts (off center)

$$\begin{bmatrix} \boxed{0} & 1 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} \boxed{1} & 0 \\ 0 & -1 \end{bmatrix}$$

- Prewitt (on center)

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & \boxed{0} & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- Sobel (on center)

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & \boxed{0} & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -2 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

## Edge Thinning

- Thresholding of gradient magnitude generally produces a thick edge.
- Edge should be thinned to produce most accurate result.

1. Set  $S = \{s : |\nabla f(s)| \geq T\}$
2. Set  $D = \emptyset$  (detected edge points)
3. For each  $s \in S$ 
  - (a) Compute  $\theta = \text{gradient direction at } s$ .
  - (b) Select out  $P = \text{all pixels in direction } \theta \text{ starting at } s \text{ within maximum distance } d_{max} \text{ from } s$ .
  - (c) If  $|\nabla f(s)| \geq \max_{p \in P} \{|\nabla f(p)|\}$ , then

$$D \leftarrow D + \{s\}$$

