EE 637 Midterm II

April 11, Spring 2007

Name: Key

Instructions:

- This is a 50 minute exam containing **three** problems.
- Each part of each problem is worth 10 points.
- You may **only** use your brain and a pencil (or pen) to complete this exam.
- You **may not** use your book, notes, or a calculator.

Good Luck.

Fact Sheet

- Function definitions

$$\mathrm{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1 - |t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFT

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \\ x(t) &= \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \end{aligned}$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X(-f)$$

$$x(t - t_0) \overset{CTFT}{\Leftrightarrow} X(f) e^{-j2\pi f t_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|} X(f/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} x(-f)$$

$$x(t) e^{j2\pi f_0 t} \overset{CTFT}{\Leftrightarrow} X(f - f_0)$$

$$x(t) y(t) \overset{CTFT}{\Leftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(f) Y(f)$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) Y^*(f) df$$

- CTFT pairs

$$\mathrm{sinc}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{rect}(f)$$

$$\mathrm{rect}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{sinc}(f)$$

For $a > 0$

$$\frac{1}{(n-1)!} t^{n-1} e^{-at} u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{(j2\pi f + a)^n}$$

- DTFT

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \end{aligned}$$

- DTFT pairs

$$a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{1 - a e^{-j\omega}}$$

- Rep and Comb relations

$$\mathrm{rep}_T [x(t)] = \sum_{k=-\infty}^{\infty} x(t - kT)$$

$$\mathrm{comb}_T [x(t)] = x(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

$$\mathrm{comb}_T [x(t)] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \mathrm{rep}_{\frac{1}{T}} [X(f)]$$

$$\mathrm{rep}_T [x(t)] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \mathrm{comb}_{\frac{1}{T}} [X(f)]$$

- Sampling and Reconstruction

$$\begin{aligned} y(n) &= x(nT) \\ Y(e^{j\omega}) &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right) \\ s(t) &= \sum_{k=-\infty}^{\infty} y(k) \delta(t - kT) \\ S(e^{j\omega}) &= Y\left(e^{j\omega T}\right) \end{aligned}$$

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Problem 1.(34pt)

You are given a printer and told that the color of each pixel is specified by the three values $[a, b, c]^t$ where

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} (255-a)^{1/\alpha} \\ (255-b)^{1/\alpha} \\ (255-c)^{1/\alpha} \end{bmatrix}.$$

a) What do the values a, b, and c represent?

b) What is the γ-value for this color coordinate system?

c) What is the white point for this color coordinate system?

d) If $255 \geq a \geq 0$, $255 \geq b \geq 0$, and $255 \geq c \geq 0$, then is it necessary to specify the values of a, b, and c in floating point, or are unsigned characters sufficient?

a). a, b. c represent the C, M, Y values.

b) $\gamma = \frac{1}{\alpha}$

c). $\begin{bmatrix} 255-a \\ 255-b \\ 255-c \end{bmatrix} = \begin{bmatrix} 255 \\ 255 \\ 255 \end{bmatrix}$ $\Rightarrow$ the write point of this color coordinate is: $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

d). Unsigned characters sufficient

Because $\begin{pmatrix} 255-a \\ 255-b \\ 255-c \end{pmatrix}$ is already the Gamma-corrected image pixel values.

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Problem 2.(33pt)

Consider an image display system where Y represents the luminance of the output light in units proportional to energy.

When the background luminance is $Y = 10$ a viewer can just notice a spot when it has a luminance of $Y_s = 10.1$, and it has a diameter of 10 degrees in units of visual subtended angle.

a) What is the just noticeable contrast?

b) What is the contrast sensitivity?

c) Assuming that Weber's Law holds true, what luminance must the spot have in order to be just noticeable when the background luminance is $Y = 1$?

d) Select a function $f(Y)$ so that equal quantization steps in the value of $f(Y)$ represent equal changes in contrast. Justify your selection.

a). $\Delta Y_{JND} = Y_s - Y = 0.1 \Rightarrow C_{JND} = \frac{\Delta Y_{JND}}{Y} = \frac{0.1}{10} = 0.01$

b). Contrast sensitivity $S = \frac{1}{C_{JND}} = 100$

c). Weber's Law says. the contrast sensitivity S is approximately independent of the background luminance.

$$\therefore S = \frac{Y}{\Delta Y_{JND}} = \text{constant} = 100$$

$\therefore$ when $Y = 1$. $\Delta Y_{JND} = 0.01 \Rightarrow Y_s = 1.01$

d).

$$\Delta(f(Y)) = \frac{\Delta Y}{Y}$$

$$\Rightarrow d(f(Y)) = \frac{1}{Y} dY$$

$$\Rightarrow \int d(f(Y)) = \int \frac{1}{Y} dY$$

$$\Rightarrow f(Y) = \log(Y)$$

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Problem 3.(33pt)

Consider the linear time-invariant discrete-time filter

$$y(n) = x(n) * h(n)$$

with input $x(n)$, output $y(n)$, and impulse response $h(n)$. Further, assume that $x(n)$ is created by sampling a continuous-time signal $s(t)$ as

$$x(n) = s(nT)$$

where $T = 1$.

a) Specify a simple FIR filter $h(n)$ so that $y(n)$ is approximately equal to $\left.\frac{ds(t)}{dt}\right|_{t=n-\frac{1}{2}}$.

b) Specify a simple FIR filter $h(n)$ so that $y(n)$ is approximately equal to $\left.\frac{ds(t)}{dt}\right|_{t=n+\frac{1}{2}}$.

c) Specify a simple FIR filter $h(n)$ so that $y(n)$ is approximately equal to $\left.\frac{d^2s(t)}{dt^2}\right|_{t=n}$.

d) Specify an operation on $y(n)$ which determines when $\frac{d^2s(t)}{dt^2} = 0$ for some value of $n \leq t \leq n+1$.

a). $\because \left.\frac{ds(t)}{dt}\right|_{t=n-\frac{1}{2}} \approx \frac{s(n)-s(n-1)}{n-(n-1)} = s(n)-s(n-1)$

$\therefore y(n) = x(n) * h(n) = s(n) * h(n) = s(n) - s(n-1)$

$\Rightarrow h(n) = \delta(n) - \delta(n-1)$

b). Similarly, $\left.\frac{ds(t)}{dt}\right|_{t=n+\frac{1}{2}} \approx \frac{s(n+1)-s(n)}{(n+1)-n} = s(n+1) - s(n)$

$\therefore y(n) = s(n) * h(n) = s(n+1) - s(n)$

$\Rightarrow h(n) = \delta(n+1) - \delta(n)$

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c). From a). b). we have

$$\left.\frac{d^2 s(t)}{dt^2}\right|_{t=n} \approx \frac{\left.\frac{ds(t)}{dt}\right|_{t=n+\frac{1}{2}} - \left.\frac{ds(t)}{dt}\right|_{t=n-\frac{1}{2}}}{(n+\frac{1}{2})-(n-\frac{1}{2})} \approx s(n+1)-s(n)-(s(n)-s(n-1))$$

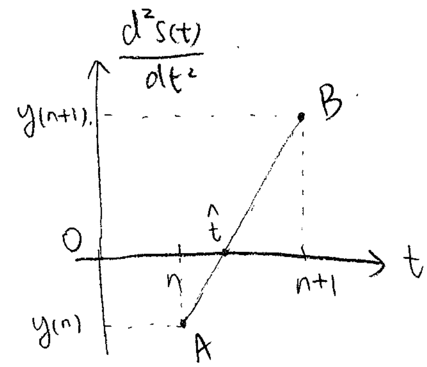
$$= s(n+1)-2s(n)+s(n-1)$$

therefore, $y(n) = s(n) * h(n) = s(n+1) - 2s(n) + s(n-1)$

$$\Rightarrow h(n) = \delta(n+1) - 2\delta(n) + \delta(n-1)$$

d). in c) we know that $y(n) \approx \left.\frac{d^2 s(t)}{dt^2}\right|_{t=n}$.

$\therefore y(n+1) \approx \left.\frac{d^2 s(t)}{dt^2}\right|_{t=n+1}$.



suppose $s(t)$ is a smooth function on $[n, n+1]$.

if $y(n) \cdot y(n+1) < 0$. i.e., different signs.

we can approximate $\frac{d^2 s(t)}{dt^2}$ as the linear interpolation of the two point $(n, y(n))$. $(n+1, y(n+1))$. for $t \in [n, n+1]$.

$$\therefore \frac{\hat{t}-n}{-y(n)} = \frac{n+1-\hat{t}}{y(n+1)} \Rightarrow \hat{t} = n + \frac{y(n)}{y(n)-y(n+1)}, \text{ s.t. } \left.\frac{d^2 s(t)}{dt^2}\right|_{t=\hat{t}} = 0$$