EE 637 Midterm I

February 23, Spring 2007

Name: Key

Instructions:

- This is a 50 minute exam containing **three** problems.
- You may **only** use your brain and a pencil (or pen) to complete this exam.
- You **may not** use your book, notes, or a calculator.

Good Luck.

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Problem 1.(33pt)

Consider the following 2D system with input $x(m,n)$ and output $y(m,n)$.

$$y(m,n) = x(m,n) + \lambda\left(x(m,n) - \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} x(m-k, n-l)\right).$$

5 a) Is this a linear system? Is this a space invariant system?

9 b) What is the 2D impulse response of this system, $h(m,n)$?

9 c) Calculate the frequency response, $H(e^{j\mu}, e^{j\nu})$?

5 d) Describe how the filter behaves when λ is positive and large.

5 e) Describe how the filter behaves when λ is negative and > -1.

a) Yes, it is a linear space invariant system

b)

$$h(m,n) = \delta(m,n)(1+\lambda) - \frac{\lambda}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1}\delta(m-k, n-l).$$

$$= \begin{cases} \left(1+\frac{8\lambda}{9}\right) & ; \quad m=n=0 \\ -\frac{\lambda}{9} & ; \quad |m|\le 1,\ |n|\le 1, \text{ but both } m,n \neq 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

(c)

$$H(e^{j\mu}, e^{j\nu}) = (1+\lambda) - \frac{\lambda}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} e^{-j\mu k}\cdot e^{-j\nu l}$$

$$= (1+\lambda) - \frac{\lambda}{9}(1+2\cos\mu)(1+2\cos\nu)$$

(d) For $\lambda > 0$ and large, the filter performs sharpening.

(e) For $-1 < \lambda < 0$, the filter performs blurring.

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Problem 2.(33pt)

Consider the causal linear space invariant system with input $x(m,n)$ and output $y(m,n)$ that is specified by

$$y(m,n) = x(m,n) + ay(m-1,n) + by(m,n-1)$$

8 a) Calculate the transfer function $H(z_1, z_2)$ for this system.

10 b) Calculate the value of

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h(m,n)$$

where $h(m,n)$ is the 2D impulse response of the system.

10 c) Calculated the value of

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h(m,n)\cos(\omega_o m)\ .$$

5 d) Is this system stable for all, none, or some values of (a,b)? Justify your answer.

(a) $$H(z_1, z_2) = \frac{1}{1 - az_1^{-1} - bz_2^{-1}} \quad //$$

(b) $$H(e^{j\mu}, e^{j\nu}) = \frac{1}{1 - ae^{-j\mu} - be^{-j\nu}} \quad ; \sum_m \sum_n h(m,n) = H(e^{j0}, e^{j0}) = \frac{1}{1-a-b} \quad //$$

(c) Let $\tilde{h}(m,n) = h(m,n)\cos(\omega_o m)$

$$\tilde{H}(e^{j\mu}, e^{+j\nu}) = \frac{1}{2} H(e^{j(\mu-\omega_o)}, e^{j\nu}) + \frac{1}{2} H(e^{j(\mu+\omega_o)}, e^{j\nu})$$

$$\therefore \sum_m \sum_n \tilde{h}(m,n) = \tilde{H}(e^{j0}, e^{j0})$$

$$= \frac{1}{2} H(e^{-j\omega_o}, e^{j0}) + \frac{1}{2} H(e^{j\omega_o}, e^{j0})$$

$$\therefore \sum_m \sum_n h(m,n)\cos(\omega_o m) = \frac{1}{2} \cdot \frac{1}{1 - ae^{j\omega_o} - b} + \frac{1}{2} \frac{1}{1 - ae^{-j\omega_o} - b} \quad //.$$

(d) System is stable for some values of a and b. For instance, it will be unstable for $a = b = 0.5$, and stable for $a = b = 0.2$. //

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Problem 3.(34pt)

Let $f(x, y)$ be a 2D image. Then the forward projection of $f(x, y)$ is given by

$$\begin{aligned} p_\theta(r) &= \mathcal{FP}\{f(x,y)\} \\ &= \int_{-\infty}^{\infty} f(r\cos(\theta) - z\sin(\theta), r\sin(\theta) + z\cos(\theta))\, dz \end{aligned}$$

and the continuous time Fourier transform of $p_\theta(t)$ is given by

$$P_\theta(\rho) = \int_{-\infty}^{\infty} p_\theta(r) e^{-j2\pi\rho r} dr$$

In order to reconstruct the image $f(x, y)$ from the projections $p_\theta(r)$, we will perform convolution back projection.

7 a) Specify the filter $H(\rho)$ so that

$$G_\theta(\rho) = H(\rho)P_\theta(\rho)$$

where $G_\theta(\rho)$ is the CTFT of the filtered projections required for perfect reconstruction.

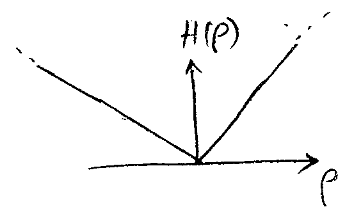
8 b) Let $\tilde{H}(\rho) = \text{rect}(\rho/2)H(\rho)$. Sketch $\tilde{H}(\rho)$, and calculate its corresponding impulse response $\tilde{h}(r)$.

7 c) Write an expression for $\tilde{g}_\theta(r)$ in terms of $\tilde{h}(r)$ and $p_\theta(r)$.

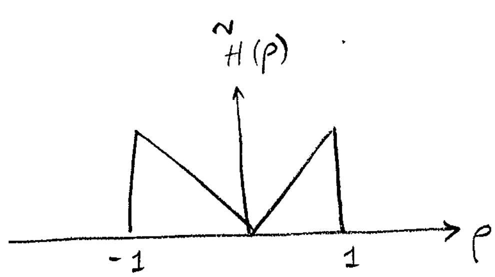
7 d) Write an expression for the reconstruction $\tilde{f}(x, y)$ in terms of the back projection of $\tilde{g}_\theta(r)$.

5 e) How do $\tilde{f}(x, y)$ and $f(x, y)$ differ?

(a) $H(\rho) = |\rho|$



(b)



$\tilde{H}(\rho) = rect(\rho/2)\, H(\rho) = rect(\rho/2) - \Lambda(\rho)$

$\therefore \tilde{h}(r) = 2\, \text{sinc}(2t) - \text{sinc}^2 t$

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(c) $\tilde{g}_\theta(r) = \tilde{h}(r) * p_\theta(r)$.

(d) $\tilde{f}(x,y) = \int_0^\pi \tilde{g}_\theta(x\cos\theta + y\sin\theta)\,d\theta$.

(e) $\tilde{f}(x,y)$ will be a blurred version of $f(x,y)$. This is because the filter $\tilde{h}(r)$, with transfer function $\tilde{H}(\rho) = |\rho| \text{ rect}(\rho/2)$, removes the high frequency components with $|\rho| > 1$.