2-D Neighborhoods

4-point neighborhood

$$\circ \times \circ \\ \circ \times \circ \\ \partial (i,j) = \{(i+1,j), (i-1,j), (i,j+1), (i,j-1)\}$$

• 8-point neighborhood

$$\partial(i,j) = \left\{ \begin{array}{c} (i+1,j), (i-1,j), (i,j+1), (i,j-1) \\ (i+1,j+1), (i-1,j+1) \\ (i+1,j-1), (i-1,j-1) \end{array} \right\}$$

- More generally, a *Neighborhood System* is any mapping with the two properties that:
 - 1. For all $s \in S$, $s \notin \partial s$
 - 2. For all $r \in S$, $r \in \partial s \Rightarrow s \in \partial r$

Boundary Conditions

- How do you process pixels on the boundary of an image??
- Consider the following example using a 4-point neighborhood

A small example image

4 neighbors of l

$$\begin{array}{ccc} & l_1 \\ l_4 & l & l_2 \\ & l_3 \end{array}$$

• Free boundary condition

$$\partial l = \{h, p, k\}$$
$$\partial p = \{l, o\}$$

• Toriodal boundary condition (asteroids)

$$\partial l = \{h, i, p, k\}$$

$$\partial p = \{l, m, d, o\}$$

• Reflective boundary condition

$$l_1 = h, l_2 = k, l_3 = p, l_4 = k$$

 $p_1 = l, p_2 = o, p_3 = l, p_4 = o$

Edge Detection

• Edges

- Edges naturally occur in images due to the discontinuities form by occlusion.
- Edges often delineate the boundaries between distinct regions.
- Edges often contain important visual and semantic information.

• Edge detection:

- The process of identifying pixels that fall along edges.
- As width any detect process subject to a trade-off between false alarm and missed detection rates.

• Performance Metrics:

- Evaluation of edge detection scemes can be difficult.
- Correct labeling of edge and non-edge pixels often requires subjective interpretation.
- Best choice of edge detection scheme usually depends on task.
- Performance metrics exist and usually use synthetic data input for evaluation.

Gradient Based Edge Detection

• Compute local estimate of gradient

$$\nabla f(x,y) = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

• From these, compute gradient magnetude and angle.

$$|\nabla f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

• Apply threshold to the magnitude of gradient

edge
$$|\nabla f| \ge T$$

no edge
$$|\nabla f| < T$$

- \bullet Choosing T
 - Too large \Rightarrow missed detections
 - Too small \Rightarrow false alarms

How to Compute Gradient

- Directional derivatives can be computed by applying a spatial filter.
- Conventional (off center)

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

• Roberts (off center)

$$\begin{bmatrix} \mathbf{0} & 1 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} \mathbf{1} & 0 \\ 0 & -1 \end{bmatrix}$$

• Prewitt (on center)

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & \boxed{0} & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

• Sobel (on center)

Edge Thining

- Thresholding of gradient magnitude generally produces a thick edge.
- Edge should be thinned to produce most accurate result.
- 1. Set $S = \{s : |\nabla f(s)| \ge T\}$
- 2. Set $D = \emptyset$ (detected edge points)
- 3. For each $s \in S$
 - (a) Compute θ = gradient direction at s.
 - (b) Select out P = all pixels in direction θ starting at s within maximum distance d_{max} from s.
 - (c) If $|\nabla f(s)| \ge \max_{p \in P} \{|\nabla f(p)|\}$, then

$$D \leftarrow D + \{s\}$$

