

1-D Rep Operation

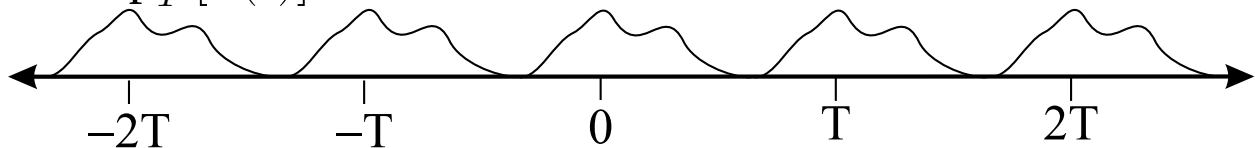
The *rep* operator periodically replicates a function with some specified period T .

$$\text{rep}_T [x(t)] = \sum_{k=-\infty}^{\infty} x(t - kT)$$

If $x(t)$ looks like



Then $\text{rep}_T [x(t)]$ looks like



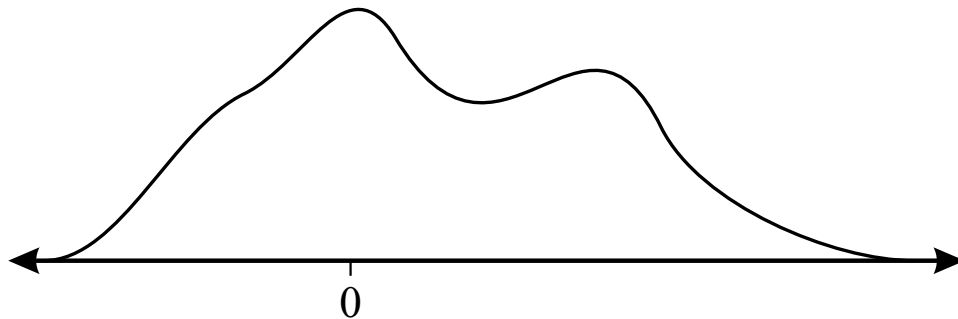
The resulting function is periodic with period T .

1-D Comb Operation

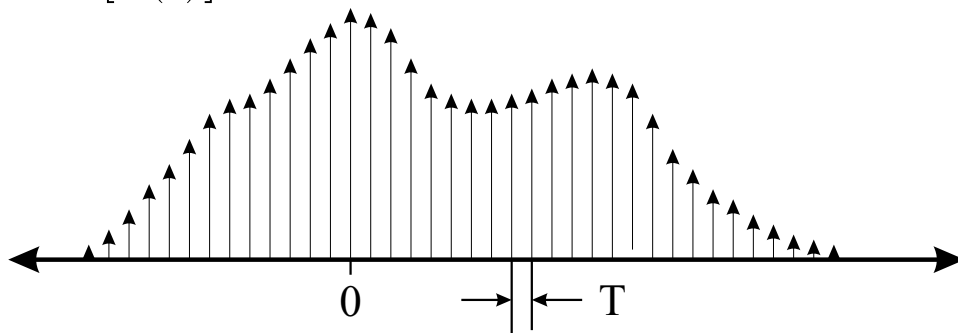
The *comb* operator multiplies a function by a periodic train of impulses.

$$\begin{aligned}\text{comb}_T [x(t)] &= \sum_{k=-\infty}^{\infty} \delta(t - kT) x(t) \\ &= x(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)\end{aligned}$$

If $x(t)$ looks like



Then $\text{comb}_T [x(t)]$ looks like



The spacing between impulses is T .

1-D Rep and Comb Transform Properties

Assume that:

$$x(t) \stackrel{CTFT}{\Leftrightarrow} X(f)$$

Then the transform relationship is:

$$\text{comb}_T [x(t)] \stackrel{CTFT}{\Leftrightarrow} \frac{1}{T} \text{rep}_{\frac{1}{T}} [X(f)]$$

$$\text{rep}_T [x(t)] \stackrel{CTFT}{\Leftrightarrow} \frac{1}{T} \text{comb}_{\frac{1}{T}} [X(f)]$$

2-D Rep and Comb Operators

2-D Rep function:

$$\begin{aligned}\text{rep}_{X,Y} [f(x, y)] \\ = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(x - mX, y - nY)\end{aligned}$$

2-D Comb function:

$$\begin{aligned}\text{comb}_{X,Y} [f(x, y)] \\ = f(x, y) \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x - mX, y - nY)\end{aligned}$$

2-D Rep and Comb Transform Properties

Assume that:

$$f(x, y) \stackrel{CSFT}{\Leftrightarrow} F(u, v)$$

Then the transform relationship is:

$$\text{comb}_{X,Y} [f(x, y)] \stackrel{CSFT}{\Leftrightarrow} \frac{1}{XY} \text{rep}_{\frac{1}{X}, \frac{1}{Y}} [F(u, v)]$$

$$\text{rep}_{X,Y} [f(x, y)] \stackrel{CSFT}{\Leftrightarrow} \frac{1}{XY} \text{comb}_{\frac{1}{X}, \frac{1}{Y}} [F(u, v)]$$