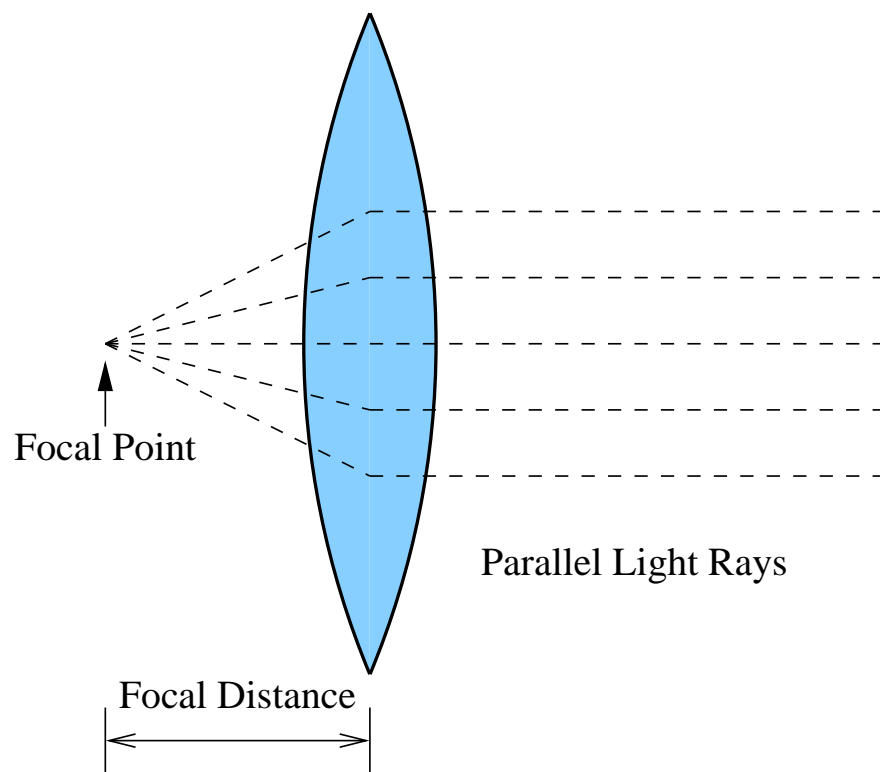


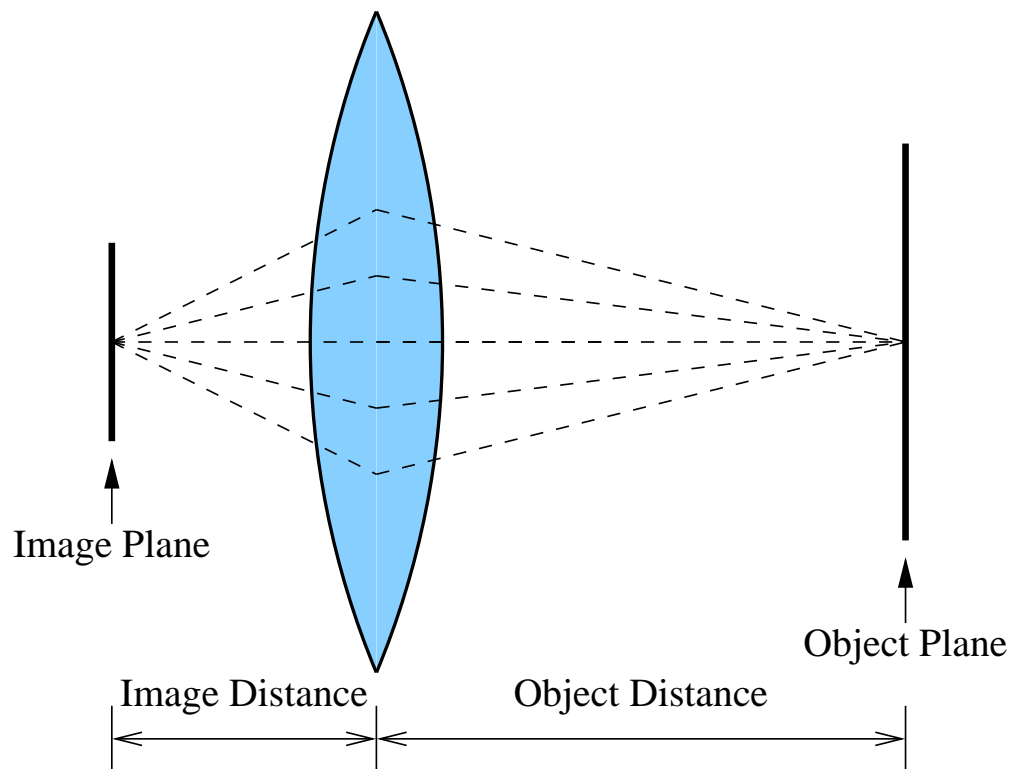
Lens: Focal Length



d_f - Focal length of lens

- Focuses incoming parallel rays of light to a point
- Based on a thin lens model

Lens: Image Formation



- Quantities:

d_f - Focal length of lens

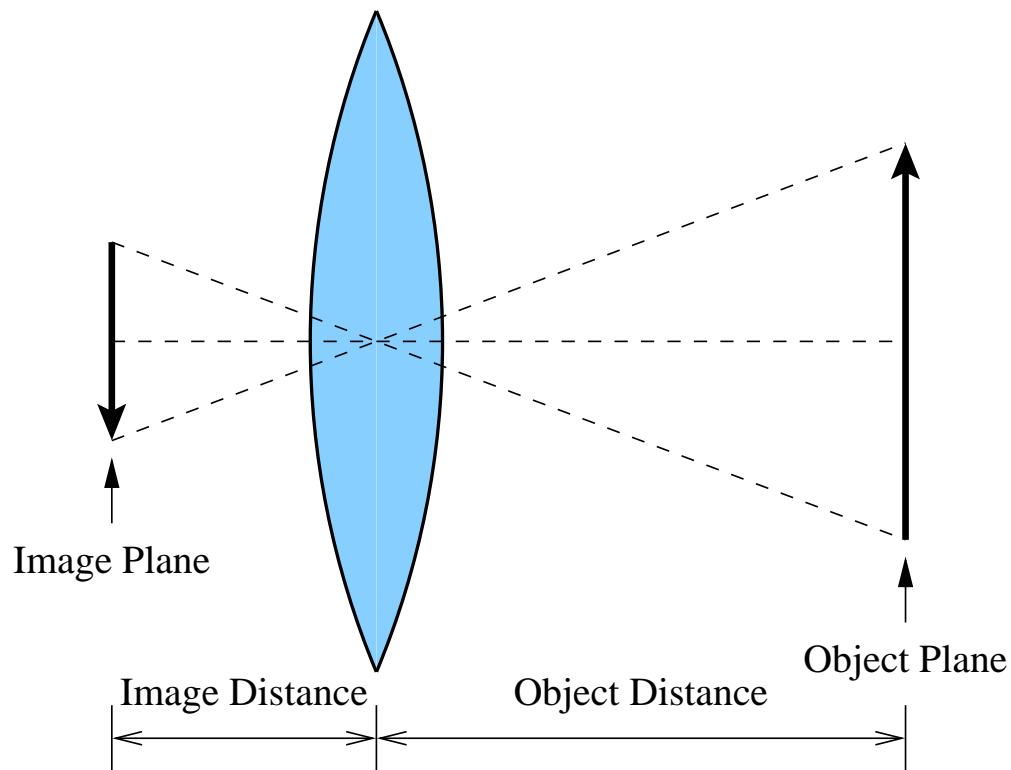
d_o - Distance to object plane

d_i - Distance to image plane

- Basic equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_f}$$

Lens: Magnification



- Quantities:

d_o - Distance to object plane

d_i - Distance to image plane

- Basic equation

$$M = -\frac{d_i}{d_o}$$

– Negative sign indicates that image is inverted

Lens: Typical Imaging Scenerios

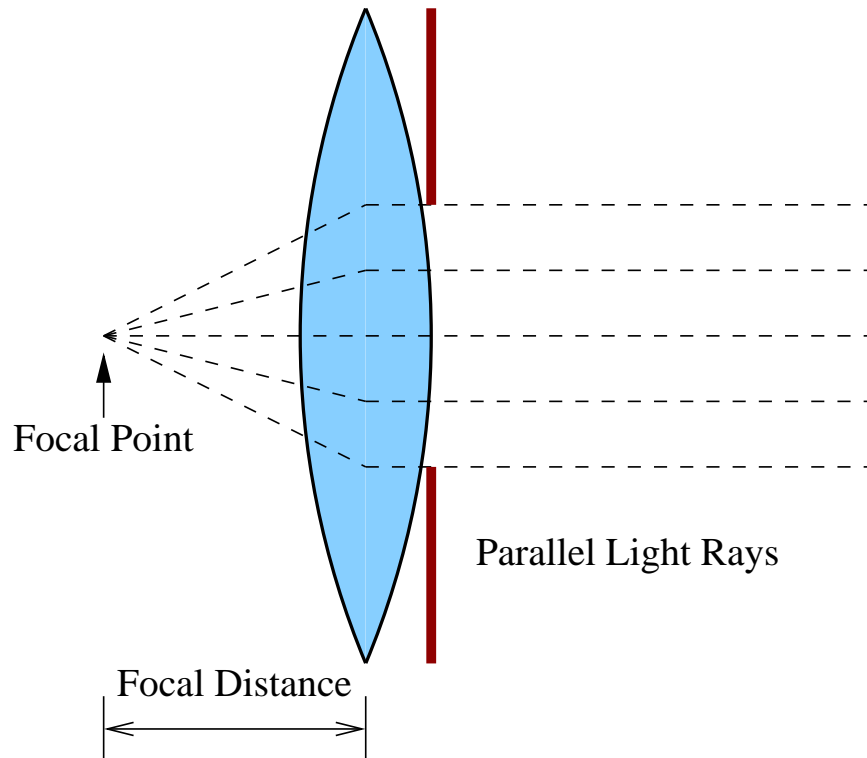
- Typical case for Photography

- $d_o \gg d_f$
- $d_i \approx d_f$
- But in addition $d_i > d_f$
- $M \ll 1$

- Typical case for microscopy

- $d_i \gg d_f$
- $d_o \approx d_f$
- But in addition $d_o > d_f$
- $M \gg 1$

Lens: Aperture and f-Stop



- Quantities:

A - Diameter of aperture

N - f-stop of lens

d_f - Focal distance of lens

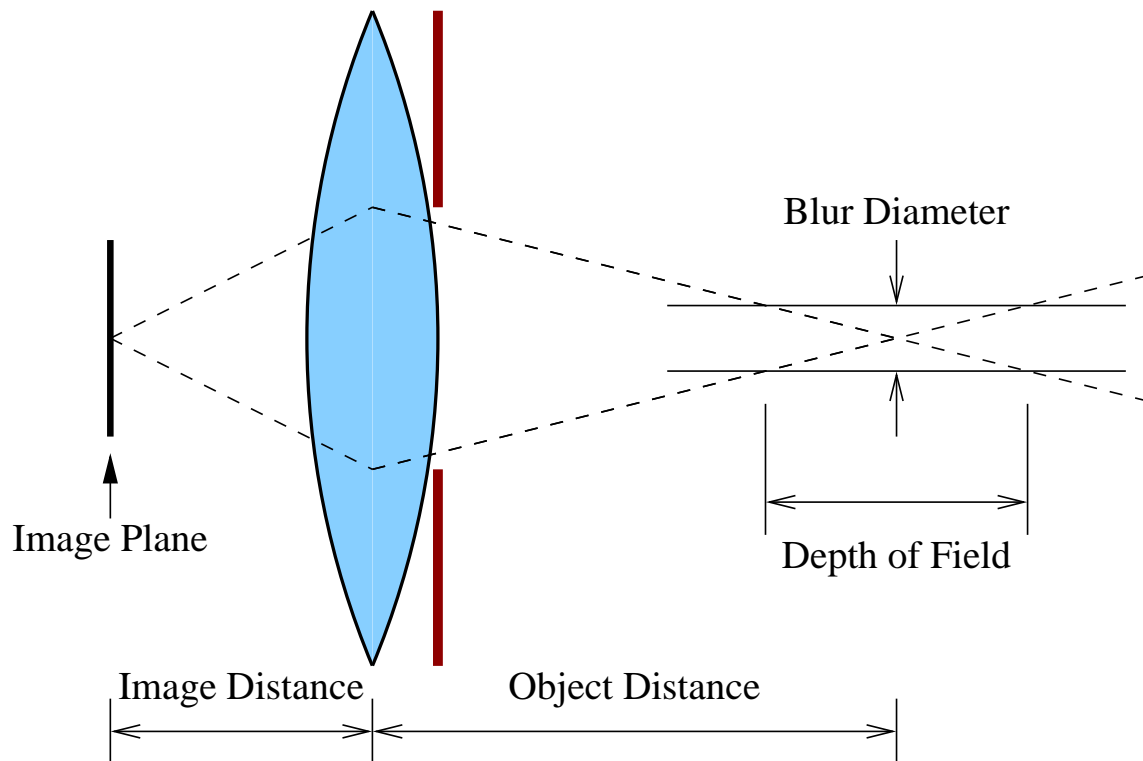
- Basic equation

$$N = \frac{d_f}{A}$$

– Large $N \Rightarrow$ small aperture \Rightarrow slow lens

– Small $N \Rightarrow$ large aperture \Rightarrow fast lens

Lens: Depth-of-Field



- Quantities:

D - depth of field

c_o - Blur diameter for object plane

N - f-stop of lens

M - Magnification

- If object is far away, then

$$\frac{D}{c_o} = \frac{2N}{-M}$$

– Small aperture increases depth-of-field

Space Domain Models for Optical Imaging Systems

- Consider an imaging system with real world image $f(x, y)$, focal plane image $g(x, y)$, and magnification M . Then the behavior of the system may be modeled as:

$$\begin{aligned} g(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi, \eta) h(x - M\xi, y - M\eta) d\xi d\eta \\ &= \frac{1}{M^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f\left(\frac{\xi}{M}, \frac{\eta}{M}\right) h(x - \xi, y - \eta) d\xi d\eta \end{aligned}$$

Define the function

$$\tilde{f}(x, y) \triangleq f\left(\frac{\xi}{M}, \frac{\eta}{M}\right)$$

- Then the imaging system act like a 2-D convolution.

$$g(x, y) = \frac{1}{M^2} h(x, y) * \tilde{f}(x, y)$$

Point Spread Functions for Optical Imaging Systems

- Definition: $h(x, y)$ is known as the *point spread function* of the imaging system.

$$g(x, y) = \frac{1}{M^2} h(x, y) * \tilde{f}(x, y)$$

- Notice that when $f(x, y) = \delta(x, y)$

$$\begin{aligned} g(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\xi, \eta) h(x - M\xi, y - M\eta) d\xi d\eta \\ &= h(x, y) \end{aligned}$$

Transfer Functions for Optical Imaging Systems

- In the frequency domain,

$$G(u, v) = \tilde{F}(u, v) \frac{1}{M^2} H(u, v)$$

$$\begin{aligned} g(x, y) &\overset{CSFT}{\longleftrightarrow} G(u, v) \\ h(x, y) &\overset{CSFT}{\longleftrightarrow} H(u, v) \\ \tilde{f}(x, y) &\overset{CSFT}{\longleftrightarrow} \tilde{F}(u, v) \end{aligned}$$

- The *Optical Transfer Function (OTF)* is

$$\frac{H(u, v)}{H(0, 0)}$$

- The *Modulation Transfer Function (MTF)* is

$$\left| \frac{H(u, v)}{H(0, 0)} \right|$$