

## **Connected Component Analysis**

- Once region boundaries have been detected, it is often useful to extract regions which are not separated by a boundary.
- Any set of pixels which is not separated by a boundary is call connected.
- Each maximal region of connected pixels is called a connected component.
- The set of connected components partition an image into segments.
- Image segmentation is an useful operation in many image processing applications.

## Connected Neighbors

- Let  $\partial s$  be a neighborhood system.
  - 4-point neighborhood system
  - 8-point neighborhood system
- Let  $c(s)$  be the set of neighbors that are connected to the point  $s$ .

For all  $s$  and  $r$ , the set  $c(s)$  must have the properties that

- $c(s) \subset \partial s$
- $r \in c(s) \Leftrightarrow s \in c(r)$

- Example:

$$c(s) = \{r \in \partial s : X_r = X_s\}$$

- Example:

$$c(s) = \{r \in \partial s : |X_r - X_s| < Threshold\}$$

- In general, computation of  $c(s)$  might be very difficult, but we won't worry about that now.

## Connected Sets

- Definition: A region  $R \subset S$  is said to be connected under  $c(s)$  if for all  $s, r \in R$  there exists a sequence of  $M$  pixels,  $s_1, \dots, s_M$  such that

$$s_1 \in c(s), s_2 \in c(s_1), \dots, s_M \in c(s_{M-1}), r \in c(s_M)$$

i.e. there is a connected path from  $s$  to  $r$ .

## Example of Connect Sets

- Consider the following image  $X_s$

```

1 1 1 0 0 0
1 1 1 0 0 0
1 1 1 0 0 0
0 0 0 1 1 1
0 0 0 1 1 1
0 0 0 1 1 1

```

$$S_1 = \{s : X_s = 1\}$$

$$S_0 = \{s : X_s = 0\}$$

- Define  $c(s) = \{r \in \partial s : X_r = X_s\}$
- Result
  - 4-point neighborhood  $\Rightarrow S_0$  and  $S_1$  are not connected sets
  - 8-point neighborhood  $\Rightarrow S_0$  and  $S_1$  **are** connected sets!

## Region Growing

- Idea - Find a connected set by growing a region from a seed point  $s_0$
- Assume that  $c(s)$  is given

$ClassLabel = 1$

Initialize  $Y_r = 0$  for all  $r \in S$

ConnectedSet( $s_0, Y, ClassLabel$ ) {

$B \leftarrow \{s_0\}$

    While  $B$  is not empty {

$s \leftarrow$  any element of  $B$

$B \leftarrow B - \{s\}$

$Y_s \leftarrow ClassLabel$

$B \leftarrow B \cup \{r : r \in c(s) \text{ and } Y_r = 0\}$

    }

    return( $Y$ )

}

## Region Growing Example (1)

The list of  
 $(i, j) \in B$   
 $(0,0)$

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 0 | 0 | 0 | 0 | 0 |
|   | 2 | 0 | 0 | 0 | 0 | 0 |
|   | 3 | 0 | 0 | 0 | 0 | 0 |
|   | 4 | 0 | 0 | 0 | 0 | 0 |

## Region Growing Example (2)

The list of  
 $(i, j) \in B$   
 $(1,0)$

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 0 | 0 | 0 | 0 |
|   | 2 | 0 | 0 | 0 | 0 | 0 |
|   | 3 | 0 | 0 | 0 | 0 | 0 |
|   | 4 | 0 | 0 | 0 | 0 | 0 |

## Region Growing Example (3)

The list of  
 $(i, j) \in B$   
 (1,1)

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 0 | 0 | 0 | 0 |
|   | 3 | 0 | 0 | 0 | 0 | 0 |
|   | 4 | 0 | 0 | 0 | 0 | 0 |

## Region Growing Example (4)

The list of  
 $(i, j) \in B$   
 (2,1)

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 0 | 0 | 0 |
|   | 3 | 0 | 0 | 0 | 0 | 0 |
|   | 4 | 0 | 0 | 0 | 0 | 0 |

## Region Growing Example (5)

The list of

$(i, j) \in B$

(3,1)

(2,2)

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 0 | 0 | 0 |
|   | 4 | 0 | 0 | 0 | 0 | 0 |

## Region Growing Example (6)

The list of

$(i, j) \in B$

(4,1)

(3,2)

(2,2)

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 0 |

## Region Growing Example (7)

The list of

$(i, j) \in B$

(3,2)

(2,2)

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 0 |

## Region Growing Example (8)

The list of  
 $(i, j) \in B$   
 (2,2)

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 0 |

## Region Growing Example (9)

The list of  
 $(i, j) \in B$   
 empty

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 0 |

## Connected Components Extraction

- Iterate through each pixel in the image.
- Extract connected set for each unlabeled pixel.

$ClassLabel = 1$

Initialize  $Y_r = 0$  for  $r \in S$

For each  $s \in S$  {

    if( $Y_s = 0$ ) {

        ConnectedSet( $s, Y, ClassLabel$ )

$ClassLabel \leftarrow ClassLabel + 1$

    }

}

# Connected Components Extraction

## Example (1)

$s = (i, j);$   
*ClassLabel*  
 $(0, 0); 1$

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 0 |

## Connected Components Extraction Example (2)

$s = (i, j);$   
*ClassLabel*  
 $(0, 1); 2$

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 2 | 2 | 2 | 2 |
|   | 1 | 1 | 1 | 2 | 2 | 2 |
|   | 2 | 0 | 1 | 1 | 2 | 2 |
|   | 3 | 0 | 1 | 1 | 2 | 2 |
|   | 4 | 0 | 1 | 2 | 2 | 0 |

## Connected Components Extraction Example (3)

$s = (i, j);$   
*ClassLabel*  
 $(2, 0); 3$

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 2 | 2 | 2 | 2 |
|   | 1 | 1 | 1 | 2 | 2 | 2 |
|   | 2 | 3 | 1 | 1 | 2 | 2 |
|   | 3 | 3 | 1 | 1 | 2 | 2 |
|   | 4 | 3 | 1 | 2 | 2 | 0 |

## Connected Components Extraction Example (4)

$s = (i, j);$   
*ClassLabel*  
 $(4, 4); 4$

The image  $X$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
|   | 1 | 1 | 1 | 0 | 0 | 0 |
|   | 2 | 0 | 1 | 1 | 0 | 0 |
|   | 3 | 0 | 1 | 1 | 0 | 0 |
|   | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation  $Y$

|   |   | j |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 2 | 2 | 2 | 2 |
|   | 1 | 1 | 1 | 2 | 2 | 2 |
|   | 2 | 3 | 1 | 1 | 2 | 2 |
|   | 3 | 3 | 1 | 1 | 2 | 2 |
|   | 4 | 3 | 1 | 2 | 2 | 4 |