Computing the Power Spectral Density From Data

$$S_x(e^{j\mu}, e^{j\nu}) = \lim_{N\to\infty} \frac{1}{(2N+1)^2} E\left[\left| \hat{X}_N(e^{j\mu}, e^{j\nu}) \right|^2 \right]$$

How do we compute expectation?

- The law of large number!

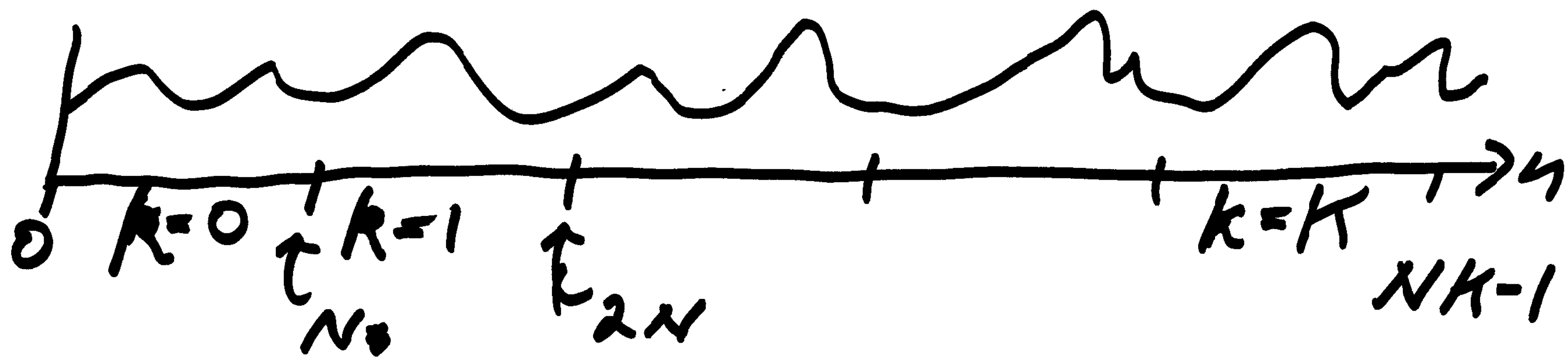
$$E[Y] = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} Y_n$$

where Y_n are i.i.d.

i.i.d. $\Leftarrow$ independent and identically distributed.

Example in 1-D

- X_n be a D-T signal for $0 \le n < NK$



- Break line into K parts, each of Length N.

k^{th} seg. $\Rightarrow Y_n^{(k)} = X_{(kN+n)}$

For each segment, compute

$$\frac{1}{N}\left|\hat{Y}^{(k)}(e^{j\omega})\right|^2$$

$$= \frac{1}{N}\Big|\underbrace{\sum_{n=0}^{N-1} Y_n^{(k)} e^{j\omega n}}_{\text{DTFT}}\Big|^2$$

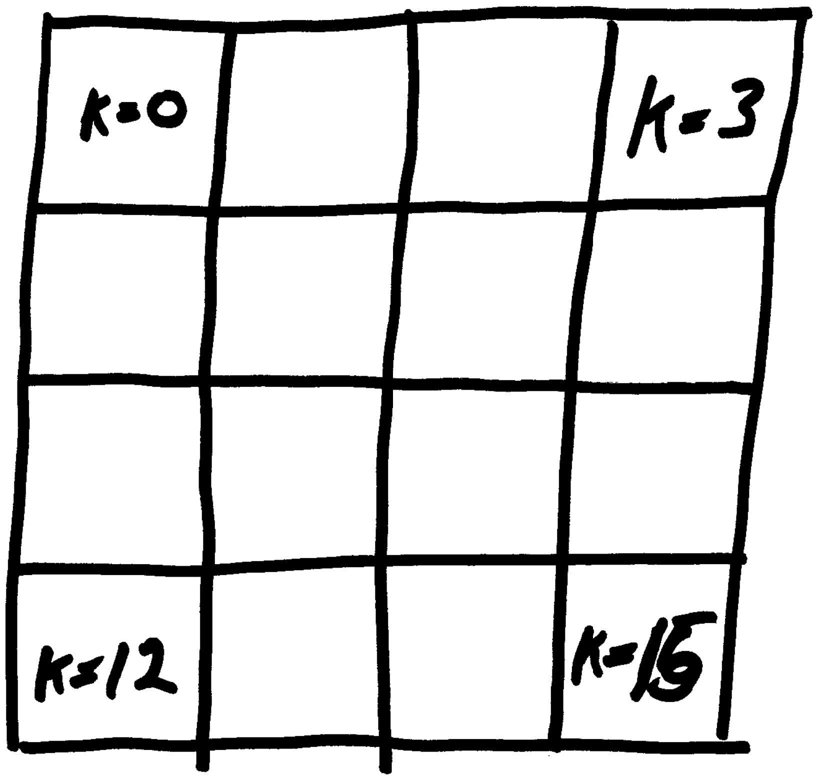
Power spectrum for $Y_n^{(k)}$.

• Average the power spectra

$$S_x(e^{j\omega}) \cong \frac{1}{NK}\sum_{k=0}^{M-1}\left|\hat{Y}^{(k)}(e^{j\omega})\right|^2$$

Example in 2-D

- Break image into K regions each with N pixels.



For each region, compute

$$\frac{1}{N}\left|\underbrace{\hat{Y}^{(k)}(e^{j\mu},e^{j\nu})}_{\text{DSFT of } y^{(k)}(m,n)}\right|^2$$

- Average to approximate expectation

$$S_x(e^{j\mu},e^{j\nu}) = \frac{1}{NK}\sum_{k=0}^{K-1}\left|\hat{Y}^{(k)}(e^{j\mu},e^{j\nu})\right|^2$$