EE 637 Final Exam

May 4, Spring 2004

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Instructions:

- Follow all instructions carefully!
- This is a 120 minute exam containing **five** problems.
- You may **only** use your brain and a pencil (or pen) to complete this exam. You **may not** use your book, notes or a calculator.

Good Luck.

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Problem 1.(15pt) Consider an image $f(x, y)$ with a CSFT given by $F(u, v)$ and a forward projection

$$\begin{aligned} p_\theta(r) &= \mathcal{FP}\{f(x,y)\} \\ &= \int_{-\infty}^{\infty} f(r\cos(\theta) - z\sin(\theta), r\sin(\theta) + z\cos(\theta))\, dz \end{aligned}$$

and let $P_\theta(\rho)$ denote the CTFT of $p_\theta(r)$.
Define the functions

$$\begin{aligned} \delta(x,y) &= \delta(x)\delta(y) \\ \mathrm{rect}(x) &= \begin{cases} 1 & \text{if } |x| \leq 1/2 \\ 0 & \text{if } |x| > 1/2 \end{cases} \end{aligned}$$

5 a) Calculate the forward projection $p_\theta(r)$ when

$$f(x, y) = \delta(x - 1, y)$$

5 b) Calculate the forward projection $p_\theta(r)$ when

$$f(x, y) = \mathrm{rect}\left(\sqrt{x^2 + y^2}\right)$$

5 c) Calculate the $P_\theta(\rho)$ (i.e. the CTFT of $p_\theta(r)$) when

$$f(x, y) = \mathrm{rect}\left(\sqrt{x^2 + y^2}\right)$$

(**Hint:** Use the fact that the CSFT of $\mathrm{rect}(\sqrt{x^2 + y^2})$ is $\mathrm{jinc}(u, v)$)

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a) $p_\theta(r) = \delta(r - \cos(\phi))$

b) $p_\theta(r) = 2\sqrt{\frac{1}{4} - r^2}\ \mathrm{rec}(r)$

c) $F(u,v) = \mathrm{jinc}(u,v)$

By the Fourier Slice theorem

$$P_\theta(\rho) = F(\rho\cos u, \rho\sin v)$$

$$= \mathrm{jinc}(\rho, 0)$$

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Problem 2.(30pt) Let $X(m,n)$ be an achromatic image taking continous values in the interval $[0,1]$, and let $T(m,n)$ be a 2-D random field of i.i.d. random variables which are uniformly distributed on the interval $[0,1]$. Let the halftoned version of $X(m,n)$ be given by

$$Y(m,n) = \begin{cases} 1 & \text{if } X(m,n) \geq T(m,n) \\ 0 & \text{if } X(m,n) < T(m,n) \end{cases}$$

+3 answer a) Is $Y(m,n)$ a stationary random process?

b) Calculate $\mu(m,n) = E\left[Y(m,n)\right]$

c) Calculate $E\left[D(m,n)D(m+k,n+l)\right]$ for $D(m,n) = Y(m,n) - \mu(m,n)$.

+3 answer d) Is $D(m,n)$ a stationary random process?

e) For the special case of $X(m,n) = g$, compute the power spectral density $S(e^{j\mu}, e^{j\nu})$ of $D(m,n)$.

+1 answer f) Is $Y(m,n)$ a good quality halftone of $X(m,n)$. Justify your answer.

Yes, full range = 2

a) No because $E[Y(m,n)]$ is a function of (m,n)

b) $E[Y(m,n)] = 1 \cdot P\{T(m,n) \leq X(m,n)\}$

$= X(m,n)$

c) $E[D(m,n)D(m+k,n+l)]$

$= \delta(k,l)\, E\left[(D(m,n))^2\right]$

$= \delta(k,l)\left[(1-X(m,n))^2 P\{T(m,n) \leq X(m,n)\} + (-X(m,n))^2 P\{T(m,n) > X(m,n)\}\right]$

$= \delta(k,l)\left[(1-X(m,n))^2 (X(m,n)) + (X(m,n))^2 (1-X(m,n))\right]$

$= \delta(k,l)\,(1-X(m,n))(X(m,n))\left[1-X(m,n) + X(m,n)\right]$

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$$= \delta(k,l)\,(1 - x(m,n))\,x(m,n)$$

d) No, because $E[(D(m,n))^2]$ is a function of (m,n)

e) In this case,

$$R(k,l) = \delta(k,l)\,(1-g)g$$

$$S(e^{j\mu}, e^{j\nu}) = (1-g)g$$

f) No because it contain too much low frequency energy.

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Problem 3.(25pt) Consider the following 2-D LSI systems. The first systems has input $x(m,n)$ and output $y(m,n)$, and the second system has input $y(m,n)$ and output $z(m,n)$.

$$\begin{aligned} y(m,n) &= x(m,n) + ay(m,n-1) && \text{S1} \\ z(m,n) &= y(m,n) + bz(m-1,n) && \text{S2} \end{aligned}$$

a) Calculate the 2-D frequency response $H_1(e^{j\mu}, e^{j\nu}) = \frac{Y(e^{j\mu},e^{j\nu})}{X(e^{j\mu},e^{j\nu})}$.
b) Calculate the 2-D frequency response for system $H_2(e^{j\mu}, e^{j\nu}) = \frac{Z(e^{j\mu},e^{j\nu})}{Y(e^{j\mu},e^{j\nu})}$.
c) Calculate the 2-D frequency response for system $H_3(e^{j\mu}, e^{j\nu}) = \frac{Z(e^{j\mu},e^{j\nu})}{X(e^{j\mu},e^{j\nu})}$.
d) Calculate a single 2-D difference equation for the system $H_3(e^{j\mu}, e^{j\nu})$.
e) For what values of a and b is the 2-D system $H_3(e^{j\mu}, e^{j\nu})$ stable?

a) $H_1(e^{j\mu}, e^{j\nu}) = \frac{1}{1 - a e^{-j\nu}}$

b) $H_2(e^{j\mu}, e^{j\nu}) = \frac{1}{1 - b e^{-j\mu}}$

c) $H_3(e^{j\mu}, e^{j\nu}) = \frac{1}{(1 - a e^{-j\nu})(1 - b e^{-j\mu})}$

d) $$\begin{aligned} z(m,n) = x(m,n) &+ a\, z(m, n-1) \\ &+ b\, z(m-1, n) \\ &+ ab\, z(m-1, n-1) \end{aligned}$$

e) $\{a, b : |a| < 1 \text{ and } |b| < 1\}$

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Problem 4.(15pt) In a color matching experiment, the three primaries R, G, B are used to match the color of a pure spectral component at wavelength λ. (Assume that the color matching allows for color to be subtracted from the reference in the standard manner described in class.)

A each wavelength λ, the matching color is given by

$$[R, G, B] \begin{bmatrix} r(\lambda) \\ g(\lambda) \\ b(\lambda) \end{bmatrix}$$

where

$$1 = \int_0^\infty r(\lambda) d\lambda$$
$$1 = \int_0^\infty g(\lambda) d\lambda$$
$$1 = \int_0^\infty b(\lambda) d\lambda$$

Further define the white point

$$W = [R, G, B] \begin{bmatrix} r_w \\ g_w \\ b_w \end{bmatrix}$$

Let $I(\lambda)$ be the light reflected from a surface.

a) Calculate (r_e, g_e, b_e) the tristimulus values for the spectral distribution $I(\lambda)$ using primaries R, G, B and an equal energy white point.

b) Calculate (r_c, g_c, b_c) the tristimulus values for the spectral distribution $I(\lambda)$ using primaries R, G, B and white point (r_w, g_w, b_w).

c) Calculate $(r_\gamma, g_\gamma, b_\gamma)$ the gamma corrected tristimulus values for the spectral distribution $I(\lambda)$ using primaries R, G, B, white point (r_w, g_w, b_w), and $\gamma = 2.2$.

a) $r_e = \int_0^\infty I(\lambda)\, r(\lambda)\, d\lambda$

$g_e = \int_0^\infty I(\lambda)\, g(\lambda)\, d\lambda$

$b_e = \int_0^\infty I(\lambda)\, b(\lambda)\, d\lambda$

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b) $$r_c = \frac{1}{r_w} \int_0^\infty I(\lambda)\, r(\lambda)\, d\lambda$$

$$g_c = \frac{1}{g_w} \int_0^\infty I(\lambda)\, g(\lambda)\, d\lambda$$

$$b_c = \frac{1}{b_w} \int_0^\infty I(\lambda)\, b(\lambda)\, d\lambda$$

c) $$r_\gamma = 255\, r_c^{1/\gamma}$$

$$g_\gamma = 255\, g_c^{1/\gamma}$$

$$b_\gamma = 255\, b_c^{1/\gamma}$$

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Problem 5.(25) Consider the discrete-time signal x_n, and let $\hat{x}_n$ be the linear causal prediction of x_n given by

$$\hat{x}_n = \sum_{k=1}^{p} a_k \, x_{n-k}$$

Let the total squared prediction error (TSPE) be given by

$$TSPE = \sum_{n=0}^{N-1} (x_n - \hat{x}_n)^2$$

a) Determine a formula for the TSPE in terms of the prediction coefficients a_k.

b) Compute an expression for the coefficients a_k that minimize the TSPE.

c) Let $y_n = x_n - \hat{x}_n$, let p be large, and let x_n be an i.i.d. random process. Describe the power spectrum of y_n. Justify you answer.

a) Let $X = \begin{bmatrix} x_0 \\ \vdots \\ x_{N-1} \end{bmatrix}$

$Z_n = [x_{n-1}, \ldots, x_{n-p}]$

$Z = \begin{bmatrix} Z_0 \\ \vdots \\ Z_{N-1} \end{bmatrix}$ $\qquad \theta = \begin{bmatrix} a_1 \\ \vdots \\ a_p \end{bmatrix}$

$TSPE = ||X - Z\theta||^2$

b) $\nabla_\theta TSPE = 0$

$(X - Z\theta)^t(-Z) = 0$

$(X^t - \theta^t Z^t)Z = 0$

$X^t Z - \theta^t Z^t Z = 0$

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$$\theta = (Z^t Z)^{-1} Z^t X$$

c) As $p \to \infty$, y_n become uncorrelated.

So $\quad S_y(e^{j\mu}, e^{j\nu}) = \sigma_y^2$

for some constant σ_y^2.