Continuous Time Fourier Transform (CTFT)

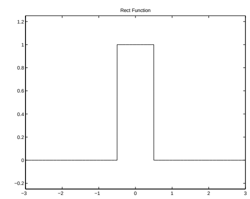
$$F(f) \;=\; \int_{-\infty}^{\infty} f(t) e^{-j2\pi ft} dt$$

$$f(t) \;=\; \int_{-\infty}^{\infty} F(f) e^{j2\pi ft} df$$

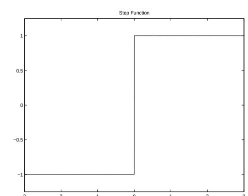
- $f(t)$ is continuous time. (Also known as continuous parameter.)
- $F(f)$ is a continuous function of frequency $-\infty < f < \infty$.

Useful Continuous Time Signal Definitions

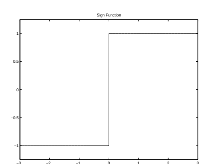
- Rect function: $\mathrm{rect}(t) = \begin{cases} 1 & \text{for } |t| \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$



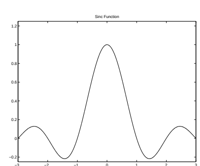
- Step function: $\mathrm{u}(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$



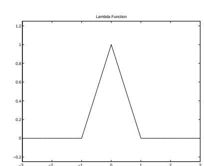
- Sign function: $\mathrm{sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t = 0 \\ -1 & \text{for } t < 0 \end{cases}$



- Sinc function: $\mathrm{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$



- Lambda function: $\Lambda(t) = \begin{cases} 1 - |t| & \text{for } |t| \leq 1 \\ 0 & \text{for } |t| > 1 \end{cases}$



Continuous Time Delta Function

- The "function" $\delta(t)$ is actually **not** a function.

- $\delta(t)$ is defined by the property that for all continuous functions $g(t)$

$$g(0) = \int_{-\infty}^{\infty} \delta(t) g(t) dt$$

- Intuitively, we may think of $\delta(t)$ as a very short pulse with unit area.

$$g(0) = \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\infty} \left[\frac{1}{\epsilon} \mathbf{rect}(t/\epsilon) \right] g(t) dt$$

Intuitively (but not rigorously)

$$\delta(t) = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \mathbf{rect}(t/\epsilon)$$

Useful CTFT Relations

$$\delta(t) \overset{CTFT}{\Leftrightarrow} 1$$

$$1 \overset{CTFT}{\Leftrightarrow} \delta(f)$$

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$\Lambda(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}^2(f)$$

CTFT Properties

Property	Time Domain Function	CTFT
Linearity	$af(t) + bg(t)$	$aF(f) + bG(f)$
Conjugation	$f^*(t)$	$F^*(-f)$
Scaling	$f(at)$	$\frac{1}{\lvert a\rvert}F(f/a)$
Shifting	$f(t - t_0)$	$\exp\{-j2\pi f t_0\} F(f)$
Modulation	$\exp\{j2\pi f_0 t\} f(t)$	$F(f - f_0)$
Convolution	f(t) * g(t)	F(f) G(f)
Multiplication	f(t) g(t)	F(f) * G(f)

- Inner product property

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) g^*(t) dt = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(f) G^*(f) df$$