EE 637 Midterm Exam #2

April16, Spring 2003

Name: ______________________________________

Instructions:

- Follow all instructions carefully!
- This is a 50 minute exam containing **three** problems.
- You may **only** use your brain and a pencil (or pen) to complete this exam. You **may not** use your book, notes or a calculator.

Good Luck.

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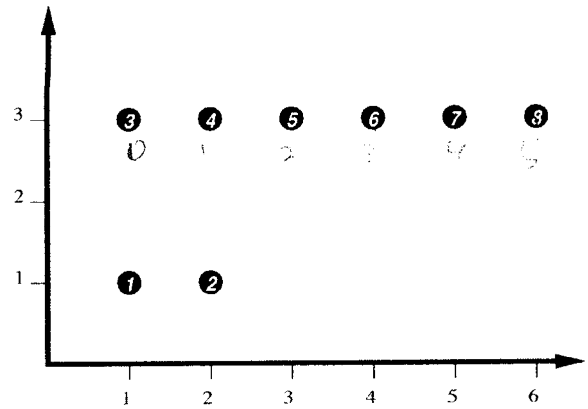
Problem 1.(33pt)



Consider the set of 7 points to be clustered in the figure shown above. The positions of the points are given by

$$v_1 = (x_1, y_1) = (1, 1)$$
$$v_2 = (x_2, y_2) = (2, 1)$$

and for $n = 0$ to 5

$$v_{(n+3)} = (x_{(n+3)}, y_{(n+3)}) = (n + n^2\epsilon, 3)$$

where ϵ is a very small positive number.

In this problem, we will investigate two different agglomerative clustering algorithms based on two different distance measures. As described in class, an agglomerative clustering algorithm works by performing pairwise merges of clusters. At each stage, the two clusters are merged that have minimum distance. This process is repeated until only a single cluster remains. The sequence of merging operations forms a binary tree with leaves consisting of individual points, and a root consisting of the set of all points.

Let k and l denote the indices of two clusters, and let S_l and S_k denote the set of points in each cluster. Then the first distance measure is given by

$$d_{k,l} = \min_{r \in S_k} \min_{s \in S_l} |v_r - v_s| \ .$$

This distance measure results in a clustering algorithm known as **minimum distance linking**.

The second distance measure is given by

$$D_{k,l} = \max_{r \in S_k} \max_{s \in S_l} |v_r - v_s| \ .$$

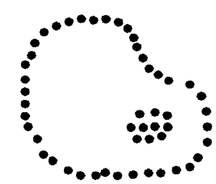
This distance measure results in a clustering algorithm known as **maximum distance linking**.

a) Consider the specific example of $S_k = \{3, 4\}$ and $S_l = \{5, 6\}$ for the points shown in the figure above. Calculate the values of $d_{k,l}$ and $D_{k,l}$.

b) Draw the binary tree resulting from minimum distance linking. Draw the root at the top and the leaves at the bottom, as done in the class notes. Label each internal node of the tree with the distance between the merged clusters. Important note~~s~~: It is not necessary to make the height of the internal nodes proportional to the distance of the merged clusters.

c) Draw the binary tree resulting from maximum distance linking, as done in part b).

d) Which method is most appropriate for separating the clusters shown in the following figure? Why?



a)

$$d_{KL} = \min_{r \in \{3,4\}} \min_{s \in \{5,6\}} |V_r - V_s|$$

$$= |V_4 - V_5| = 1 + (4\varepsilon - 1\varepsilon)$$

$$= 1 + 3\varepsilon$$

$$D_{KL} = \max_{r \in \{3,4\}} \max_{s \in \{5,6\}} |V_r - V_s|$$

$$= |V_3 - V_6|$$

$$= 3 + 9\varepsilon$$

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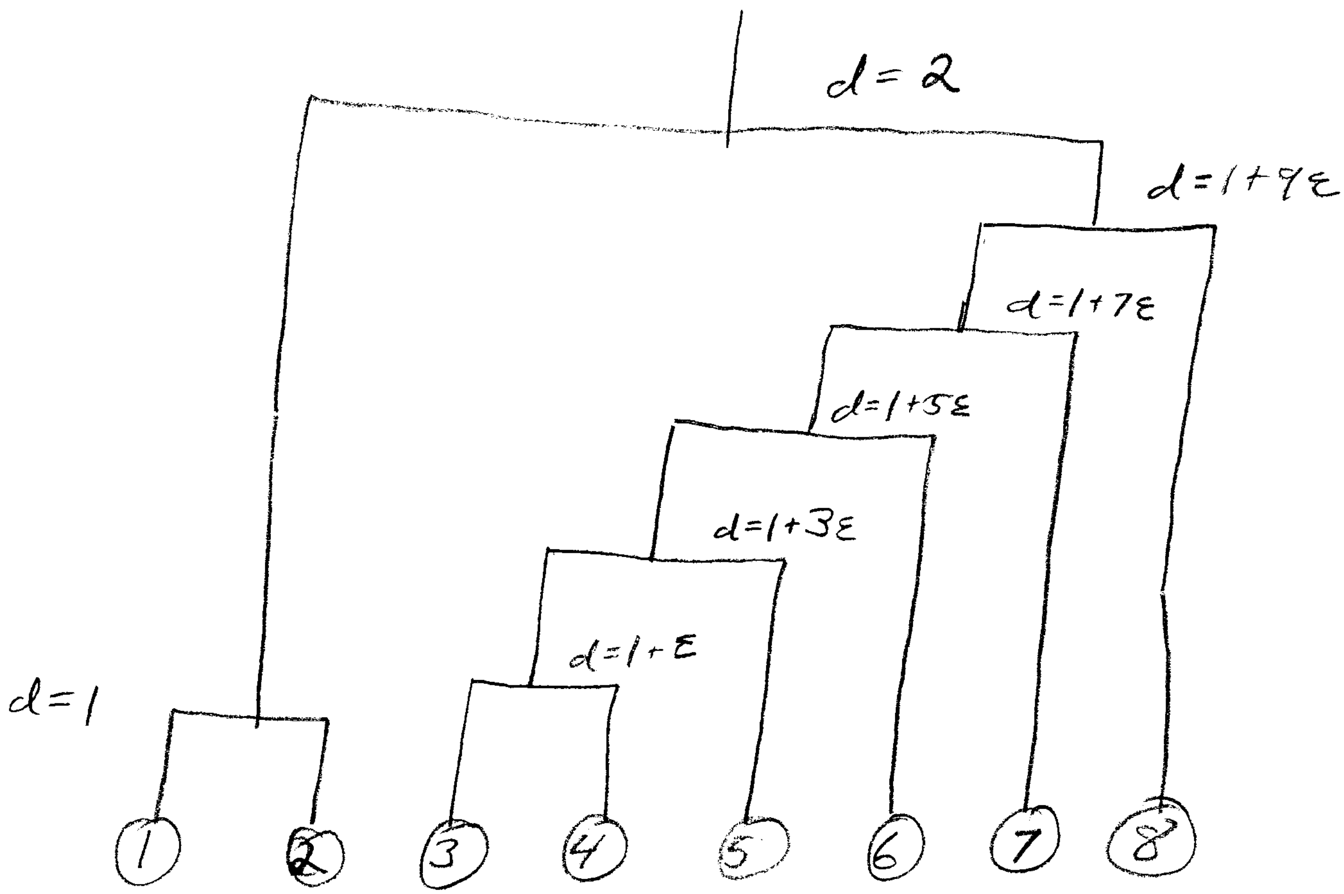
b)



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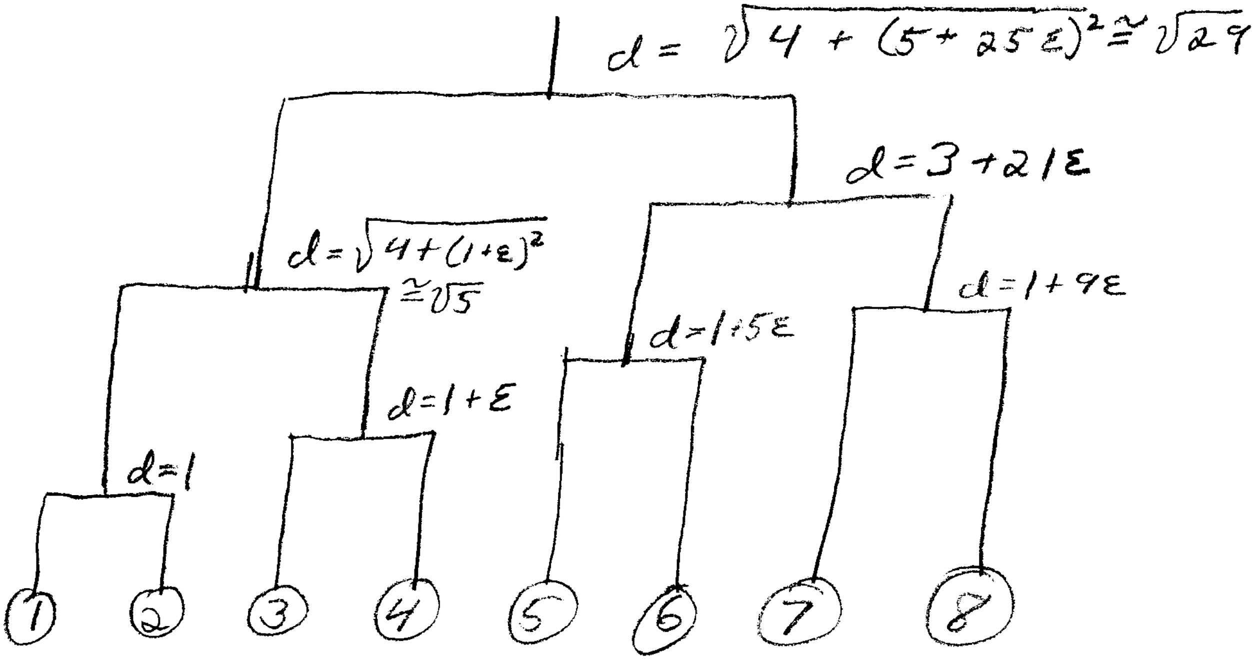
c)



d) Minimum pixel linking is most appropriate because it links together nearby points to form two clusters with one cluster contained inside the other.

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Problem 2.(34pt)

Consider the emissive display device which is accurately modeled by the equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} r^\alpha \\ g^\alpha \\ b^\alpha \end{bmatrix}$$

where r, g, and b are the red, green, and blue inputs in the range 0 to 255.

a) What is the gamma of the device?

b) What are the chromaticity components (x_w, y_w) of the device's white point.

c) What are the chromaticity components (x_r, y_r), (x_g, y_g), and (x_b, y_b) of the device's three primaries.

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Problem 2.(34pt)

Consider the emissive display device which is accurately modeled by the equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} r^\alpha \\ g^\alpha \\ b^\alpha \end{bmatrix}$$

a) What is the gamma of the device?

b) What are the chromaticity components (x, y) of the device's white point.

c) What are the chromaticity components (x, y) of the device's primaries.

a) $\gamma = \alpha$

b) $\left(\frac{a+b+c}{(d+b+c+d+e+f+g+h+i)}, \frac{d+e+f}{(a+b+c+d+e+f+g+h+i)} \right)$

c) $(x_r, y_r) = \left(\frac{d}{a+b+g}, \frac{b}{a+b+g} \right)$

$(x_g, y_g) = \left(\frac{b}{b+e+h}, \frac{e}{b+e+h} \right)$

$(x_b, y_b) = \left(\frac{c}{c+f+i}, \frac{f}{c+f+i} \right)$

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Problem 3.(33pt)

In the following problem, we assume that spectral light measurements are discretized into 31 component vectors ranging from 300 nm to 700 nm in 10 nm steps. Using this assumption, the light reflected from an object has the spectrum

$$I_i = R_i S_i$$

where $1 \leq i \leq 31$ and S_i is the source illumination, R_i is the surface reflectance, and I_i is the reflected light. Further define, x_i, y_i, and z_i as the color matching functions for the X, Y, Z tristimulus values.

For documents printed on a *PurdueJet* printer, it is known that the spectral surface reflectance is given by

$$R = \begin{bmatrix} R_1 \\ \vdots \\ R_{31} \end{bmatrix} = \mathbf{A} \begin{bmatrix} c \\ m \\ y \end{bmatrix}$$

where the columns of the matrix $\mathbf{A}$ are the spectral reflectance's of the cyan, magenta, and yellow inks respectively.

a) Calculate a equation for the X, Y, Z components of the reflected light.

b) In general, is it possible for two different surface reflectance functions $R^{(1)}$ and $R^{(2)}$ to have the same X, Y, Z components? Characterize the space of possible spectral differences $\Delta R = R^{(1)} - R^{(2)}$ that will result in no change of the X, Y, Z components.

c) For documents printed on a *PurdueJet* printer, calculate an expression for the vector $[c, m, y]^t$ as a function of the measured value of $[X, Y, Z]^t$ and the known illuminant S. (Hint: You will need to define matrices in terms of the color matching functions and the known illuminant.)

c) For documents printed on a *PurdueJet* printer, calculate an expression for the spectral reflectance vector R as a function of the measured value of $[X, Y, Z]^t$ and the known illuminant S.

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a)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \begin{bmatrix} I_i \end{bmatrix} = \begin{bmatrix} \sum_i x_i I_i \\ \sum_i y_i I_i \\ \sum_i z_i I_i \end{bmatrix}$$

define the matix

$$B_{ij} = \begin{cases} x_j & \text{for } i=1 \\ y_j & \text{for } i=2 \\ z_j & \text{for } i=3 \end{cases}$$

and the vector $I = \begin{bmatrix} I_1 \\ \vdots \\ I_{31} \end{bmatrix}$

then

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = B I$$

b) Yes.

Define $D = diag\{s_1, \ldots, s_{31}\}$

Then

$$I = D R$$

where $R = \begin{bmatrix} R_1 \\ \vdots \\ R_{31} \end{bmatrix}$

So we know that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = B D R$$

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(BD) is a 3x31 matrix

So we know that

$$BD\,\Delta R = 0$$

$\Rightarrow$ ΔR is in the null space of (BD)

$\Rightarrow$ ΔR falls in a 28 dimensional subspace

c)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = BDR$$

$$= BDA \begin{bmatrix} c \\ m \\ y \end{bmatrix}$$

$$\begin{bmatrix} c \\ m \\ y \end{bmatrix} = [BDA]^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

BCA is a 3x3 matrix

d)

$$R = A \begin{bmatrix} c \\ m \\ y \end{bmatrix} = A [BDA]^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$