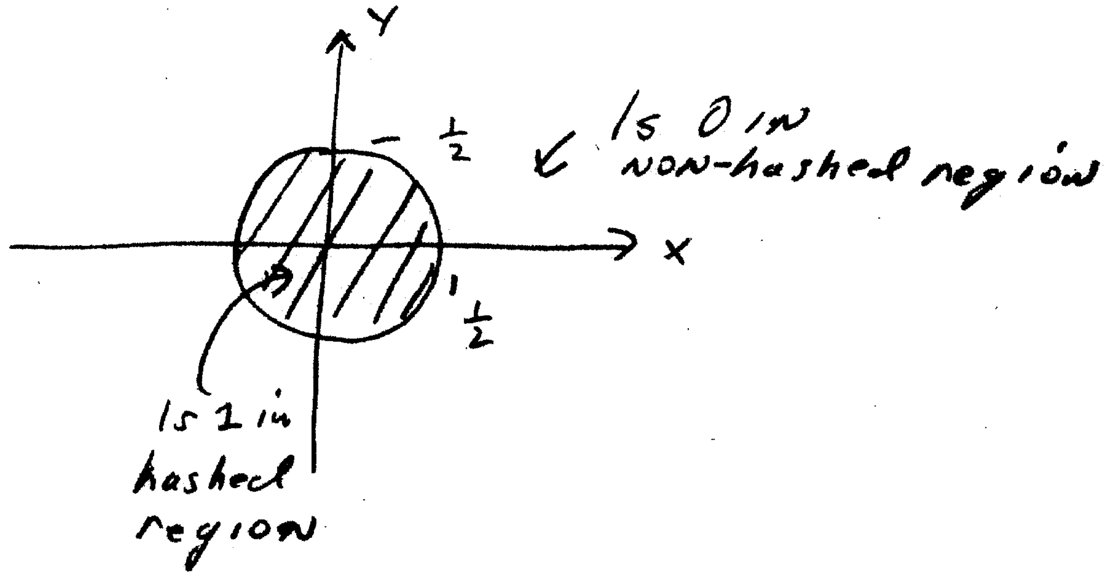
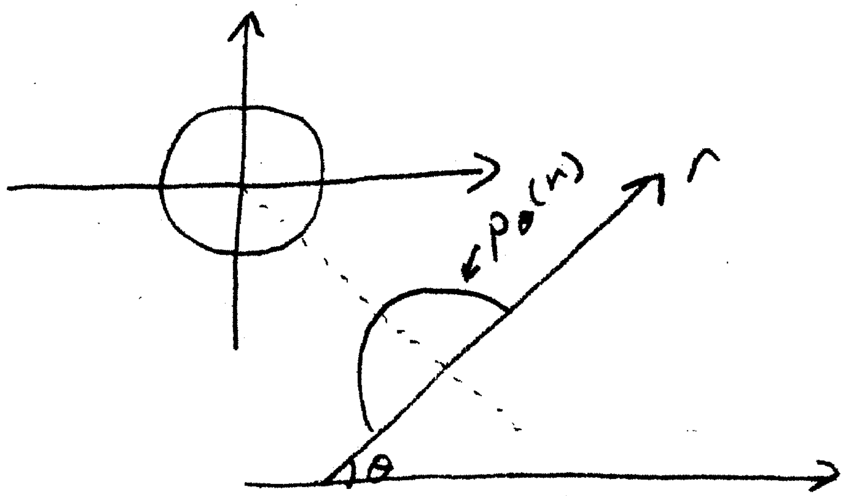
EE 637 Exam #1
Solution

a)

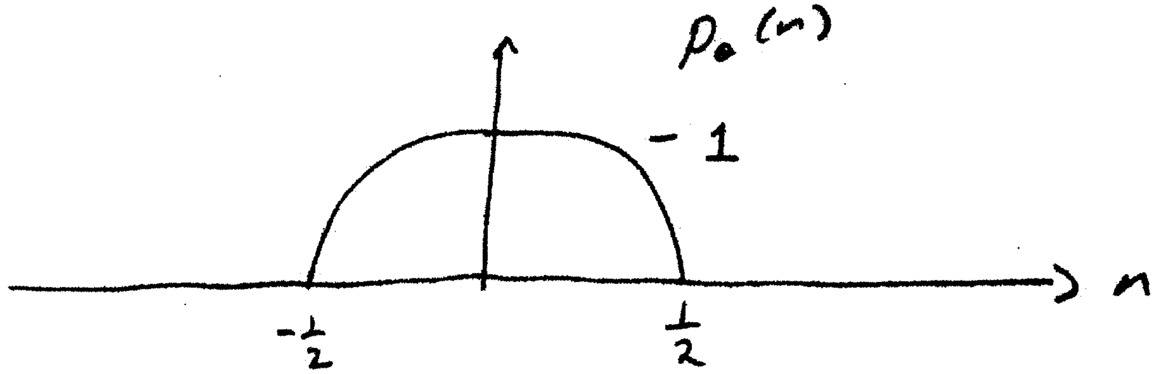


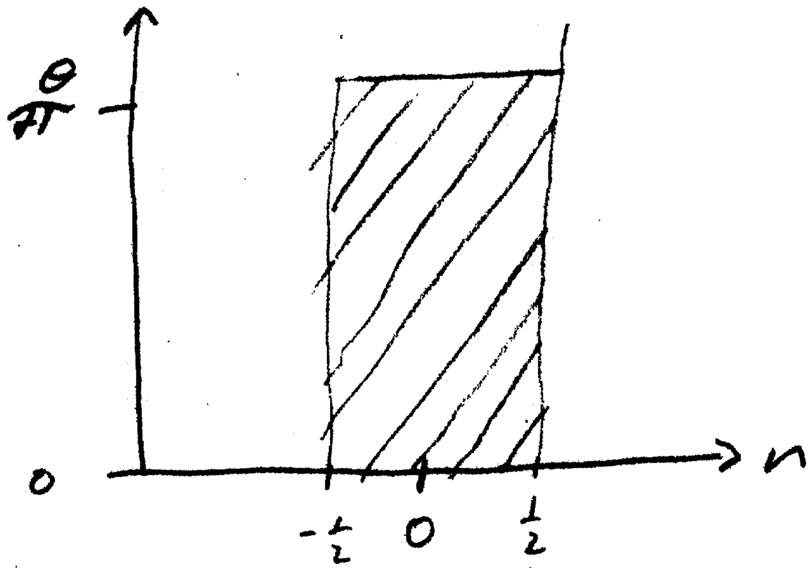
b)



$$p_\theta(r) = \begin{cases} 2\int_0^{\sqrt{(\frac{1}{2})^2 - r^2}} 1 \, dz & \text{for } |r| < \frac{1}{2} \\ 0 & \text{for } |r| > \frac{1}{2} \end{cases}$$

$$p_\theta(r) = 2\sqrt{\frac{1}{4} - r^2} \; \mathrm{rect}(r)$$

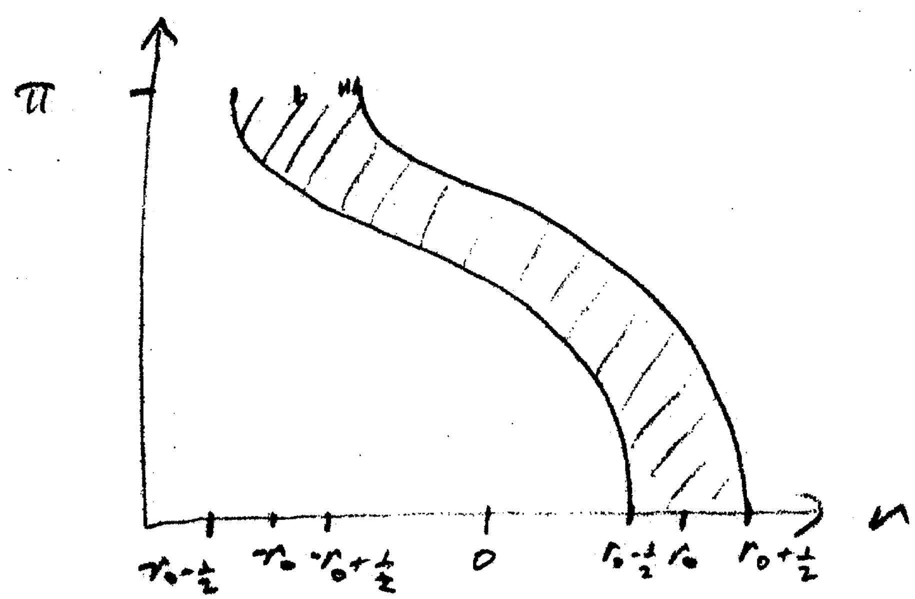




c)

$$p'_\theta(r) = p_0\left(r - r_0\cos(\theta-\varphi)\right)$$

$$= 2\sqrt{\frac{1}{4} - \left(r - r_0\cos(\theta-\varphi)\right)^2}\ \mathrm{rect}\left(r - r_0\cos(\theta-\varphi)\right)$$



d) $F(\mu, \nu) = CSFT\{circ(x,y)\}$

$$= jinc(\mu, \nu)$$

By the Fourier Slice Theorem

$$P_\theta(\rho) = F(\rho\cos\theta, \rho\sin\theta)$$

$$= jinc(\rho\cos\theta, \rho\sin\theta)$$

2)

a) $y(n) = x(n) + a\,y(n-1)$

b) Yes because Linear space invariant filters can be commuted. That is, for two 2-D filter $h(n)$ and $g(n)$ we know that

$$x(n) * h(n) * g(n)$$

$$= x(n) * g(n) * h(n)$$

c) $$G(z_1, z_2) = \frac{1}{1-az_1^{-1}} \frac{1}{1-az_2^{-1}} F(z_1, z_2)$$

d) $$(1-az_1^{-1})(1-az_2^{-1})\,G(z_1,z_2) = F(z_1, z_2)$$

$$(1 - az_1^{-1} - az_2^{-1} + a^2 z_1^{-1} z_2^{-1})\,G(z_1,z_2) = F(z_1,z_2)$$

$$G(z_1,z_2) = F(z_1,z_2) + (az_1^{-1} + az_2^{-1} - az_1^{-1}z_2^{-1})\,G(z_1,z_2)$$

$$g(m,n) = f(m,n) + a g(m-1,n) + a g(m,n-1) - a^2 g(m-1,n-1)$$

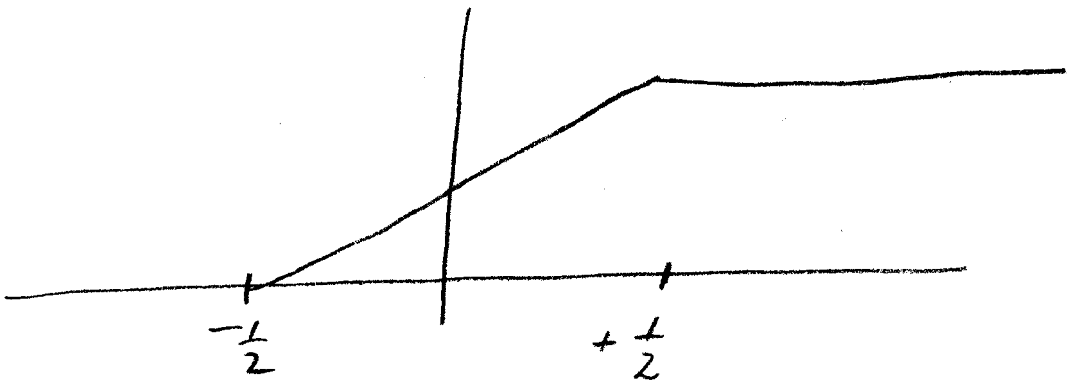
e) The 2 1-D filter only require 2 multiplies per output point, but the 2-D filter requires 3 multiplies per output point. This computational savings grows as the filter size grows.

3.)

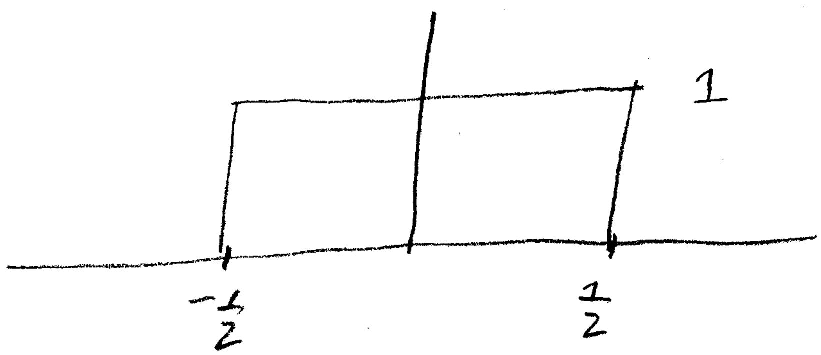
a)

$$F(x) = P\{X(m,n) \le x\}$$

$$= \begin{cases} x + \frac{1}{2} & \text{for } |x| \le \frac{1}{2} \\ 0 & \text{for } x < \frac{1}{2} \\ 1 & \text{for } x > \frac{1}{2} \end{cases}$$



$$p(x) = \begin{cases} 1 & \text{for } |x| \le \frac{1}{2} \\ 0 & \text{for } |x| > \frac{1}{2} \end{cases}$$



b) $$E[x(m,n)] = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \times dx = 0$$

for $(k,l) \neq (0,0)$

$$E[x(m,n)\, x(m+k, n+l)]$$
$$= E[x(m,n)]\, E[x(m+k, n+l)]$$
$$= 0 \cdot 0 = 0$$

for $(k,l) = (0,0)$

$$E[x(m,n)\, x(m+k, n+l)]$$
$$= E[(x(m,n))^2]$$
$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2\, dx$$
$$= 2 \int_0^{\frac{1}{2}} x^2\, dx$$
$$= 2\, \frac{1}{3} x^3 \Big|_0^{\frac{1}{2}}$$
$$= \frac{2}{3} \frac{1}{8} = \frac{1}{12}$$

$$R(k,l) = \delta(k,l)\, \frac{1}{12}$$

c) $S_x(\mu,\nu) = DSFT\left\{ \frac{1}{12}\delta(k,l) \right\}$

$$= \frac{1}{12}$$

d) $S_y(\mu,\nu) = \frac{1}{\left|1 - a e^{-j\mu}\right|^2} S_x(\mu,\nu)$

$$= \frac{1}{1 - 2a\cos(\mu) + a^2} \frac{1}{12}$$

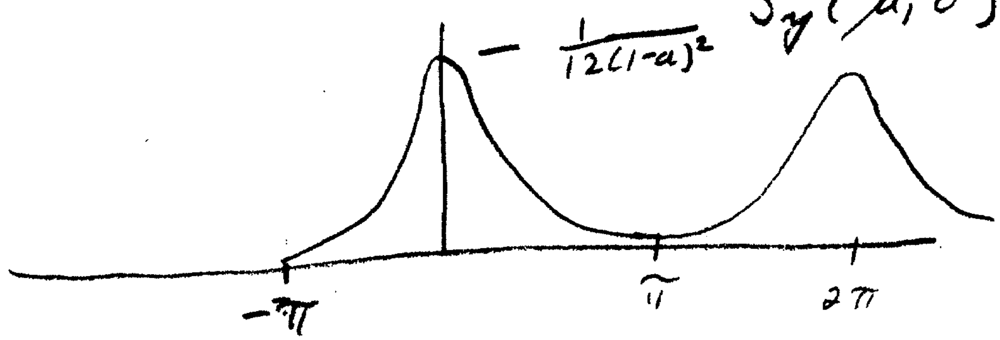
$$= \frac{1}{1 + a^2 - 2a\cos(\mu)} \frac{1}{12}$$

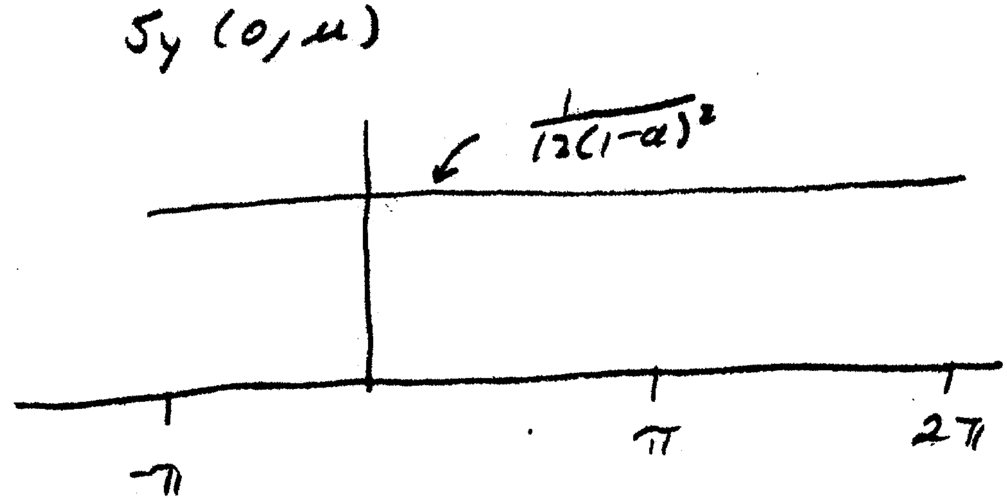
$$= \frac{1}{12(1+a^2)} \frac{1}{1 - \frac{2a}{1+a^2}\cos(\mu)}$$

e) $S_y(0,0) = \frac{1}{12(1+a^2)} \frac{1}{1 - \frac{2a}{1+a^2}}$

$$= \frac{1}{12(1 + a^2 - 2a)}$$

$$= \frac{1}{12(1-a)^2}$$





f) No, because $X(m,n)$ is NOT Gaussian

g) Yes, because it is generated applying a linear space invariant filter to a stationary process $X(m,n)$