2-D Neighborhoods

- 4-point neighborhood

$$\begin{array}{ccc} & \circ & \\ \circ & \times & \circ \\ & \circ & \end{array}$$

$$\partial(i,j) = \{(i+1,j),(i-1,j),(i,j+1),(i,j-1)\}$$

- 8-point neighborhood

$$\begin{array}{ccc} \circ & \circ & \circ \\ \circ & \times & \circ \\ \circ & \circ & \circ \end{array}$$

$$\partial(i,j) = \left\{ \begin{array}{c} (i+1,j),(i-1,j),(i,j+1),(i,j-1) \\ (i+1,j+1),(i-1,j+1) \\ (i+1,j-1),(i-1,j-1) \end{array} \right\}$$

- More generally, a *Neighborhood System* is any mapping with the two properties that:
  1. For all $s \in S$, $s \notin \partial s$
  2. For all $r \in S$, $r \in \partial s \Rightarrow s \in \partial r$

Boundary Conditions

- How do you process pixels on the boundary of an image??
- Consider the following example using a 4-point neighborhood

A small example image

$$\begin{matrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{matrix}$$

4 neighbors of l

$$\begin{matrix} & l_1 & \\ l_4 & l & l_2 \\ & l_3 & \end{matrix}$$

- Free boundary condition

$$\begin{aligned} \partial l &= \{h, p, k\} \\ \partial p &= \{l, o\} \end{aligned}$$

- Toriodal boundary condition (asteroids)

$$\begin{aligned} \partial l &= \{h, i, p, k\} \\ \partial p &= \{l, m, d, o\} \end{aligned}$$

- Reflective boundary condition

$$l_1 = h,\ l_2 = k,\ l_3 = p,\ l_4 = k$$
$$p_1 = l,\ p_2 = o,\ p_3 = l,\ p_4 = o$$

Edge Detection

- Edges
  - Edges naturally occur in images due to the discontinuities form by occlusion.
  - Edges often delineate the boundaries between distinct regions.
  - Edges often contain important visual and semantic information.
- Edge detection:
  - The process of identifying pixels that fall along edges.
  - As width any detect process subject to a trade-off between false alarm and miss detection rates.
- Performance Metrics:
  - Evaluation of edge detection scemes can be difficult.
  - Correct labeling of edge and non-edge pixels often requires subjective interpretation.
  - Best choice of edge detection scheme usually depends on task.
  - Performance metrics exist and usually use synthetic data input for evaluation.

Gradient Based Edge Detection

- Compute local estimate of gradient

$$\bigtriangledown f(x,y) = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- From these, compute gradient magnetude and angle.

$$|\bigtriangledown f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- Apply threshold to the magnitude of gradient

$$\text{edge} \quad |\bigtriangledown f| \geq T$$

$$\text{no edge} \quad |\bigtriangledown f| < T$$

- Choosing T
  - Too large $\Rightarrow$ missed detections
  - Too small $\Rightarrow$ false alarms

How to Compute Gradient

- Directional derivatives can be computed by applying a spatial filter.

- Conventional (off center)

$$\begin{bmatrix} \boxed{\text{-1}} & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} \boxed{\text{-1}} & 0 \\ 1 & \end{bmatrix}$$

- Roberts (off center)

$$\begin{bmatrix} \boxed{0} & 1 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} \boxed{1} & 0 \\ 0 & -1 \end{bmatrix}$$

- Prewitt (on center)

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & \boxed{0} & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- Sobel (on center)

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & \boxed{0} & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -2 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Edge Thining

- Thresholding of gradient magnitude generally produces a thick edge.
- Edge should be thinned to produce most accurate result.

1. Set $S = \{s : |\bigtriangledown f(s)| \geq T\}$
2. Set $D = \emptyset$ (detected edge points)
3. For each $s \in S$
   (a) Compute $\theta =$ gradient direction at s.
   (b) Select out $P =$ all pixels in direction θ starting at s within maximum distance d_{max} from s.
   (c) If $|\bigtriangledown f(s)| \geq \max_{p \in P} \{|\bigtriangledown f(p)|\}$, then

$$D \leftarrow D + \{s\}$$