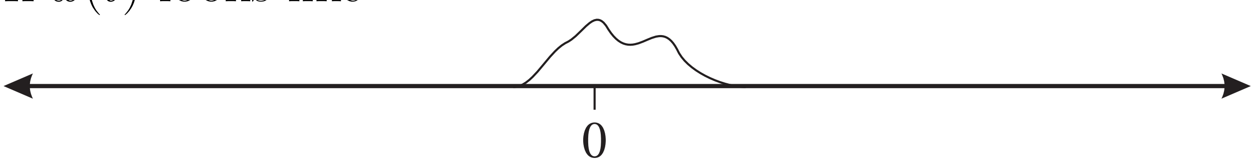
1-D Rep Operation

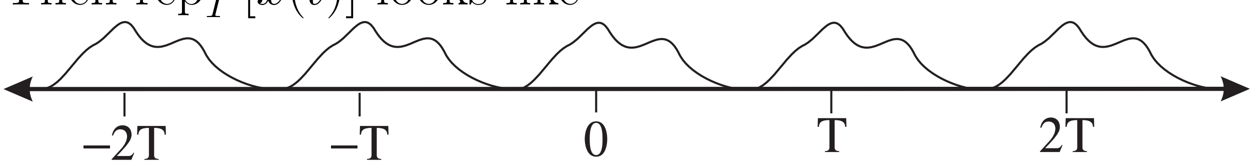
The *rep* operator periodically replicates a function with some specified period T.

$$\text{rep}_T\left[x(t)\right] = \sum_{k=-\infty}^{\infty} x(t - kT)$$

If $x(t)$ looks like



Then $\text{rep}_T\left[x(t)\right]$ looks like



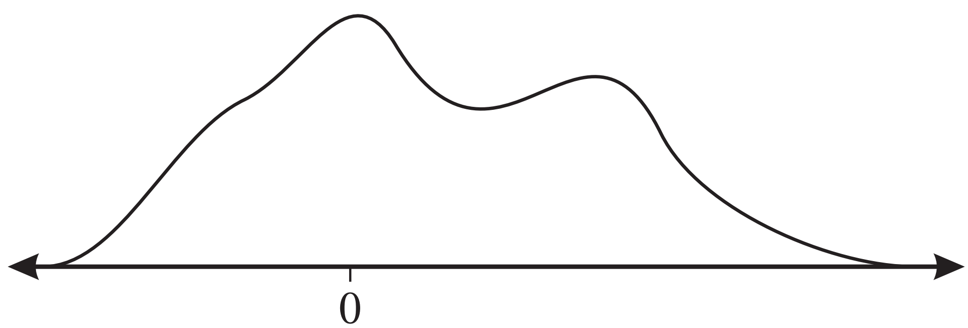
The resulting function is periodic with period T.

1-D Comb Operation

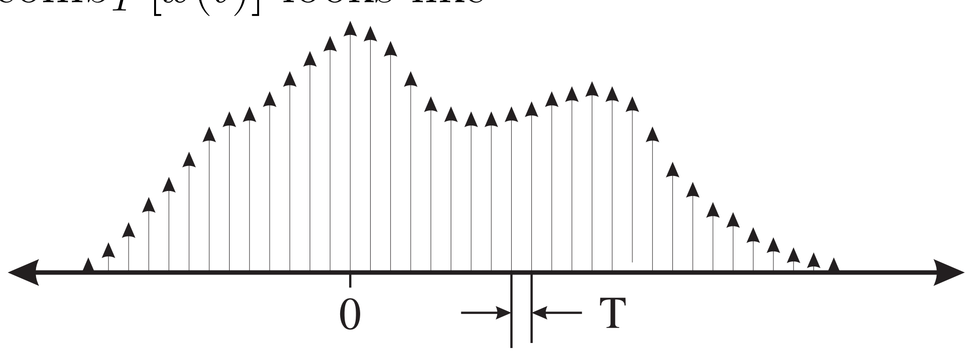
The *comb* operator multiplies a function by a periodic train of impulses.

$$\begin{aligned}\text{comb}_T\left[x(t)\right] &= \sum_{k=-\infty}^{\infty} \delta(t-kT)x(t) \\ &= x(t)\sum_{k=-\infty}^{\infty} \delta(t-kT)\end{aligned}$$

If $x(t)$ looks like



Then $\text{comb}_T\left[x(t)\right]$ looks like



The spacing between impulses is T.

1-D Rep and Comb Transform Properties

Assume that:

$$x(t) \overset{CTFT}{\Leftrightarrow} X(f)$$

Then the transform relationship is:

$$\text{comb}_T\left[x(t)\right] \quad \overset{CTFT}{\Leftrightarrow} \quad \frac{1}{T}\text{rep}_{\frac{1}{T}}\left[X(f)\right]$$

$$\text{rep}_T\left[x(t)\right] \quad \overset{CTFT}{\Leftrightarrow} \quad \frac{1}{T}\text{comb}_{\frac{1}{T}}\left[X(f)\right]$$

2-D Rep and Comb Operators

2-D Rep function:

$$\mathrm{rep}_{X,Y}\left[f(x,y)\right]$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(x - mX, y - nY)$$

2-D Comb function:

$$\mathrm{comb}_{X,Y}\left[f(x,y)\right]$$

$$= f(x,y) \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x - mX, y - nY)$$

2-D Rep and Comb Transform Properties

Assume that:

$$f(x,y) \overset{CSFT}{\Leftrightarrow} F(u,v)$$

Then the transform relationship is:

$$\mathrm{comb}_{X,Y}\left[f(x,y)\right] \quad \overset{CSFT}{\Leftrightarrow} \quad \frac{1}{XY}\mathrm{rep}_{\frac{1}{X},\frac{1}{Y}}\left[F(u,v)\right]$$

$$\mathrm{rep}_{X,Y}\left[f(x,y)\right] \quad \overset{CSFT}{\Leftrightarrow} \quad \frac{1}{XY}\mathrm{comb}_{\frac{1}{X},\frac{1}{Y}}\left[F(u,v)\right]$$