

## Connected Component Analysis

- Once region boundaries have been detected, it is often useful to extract regions which are not separated by a boundary.
- Any set of pixels which is not separated by a boundary is called connected.
- Each maximal region of connected pixels is called a connected component.
- The set of connected components partition an image into segments.
- Image segmentation is an useful operation in many image processing applications.

## Connected Neighbors

- Let  $\partial s$  be a neighborhood system.
  - 4-point neighborhood system
  - 8-point neighborhood system
- Let  $c(s)$  be the set of neighbors that are connected to the point  $s$ .

For all  $s$  and  $r$ , the set  $c(s)$  must have the properties that

- $c(s) \subset \partial s$
- $r \in c(s) \Leftrightarrow s \in c(r)$

- Example:

$$c(s) = \{r \in \partial s : X_r = X_s\}$$

- Example:

$$c(s) = \{r \in \partial s : |X_r - X_s| < \text{Threshold}\}$$

- In general, computation of  $c(s)$  might be very difficult, but we won't worry about that now.

## Connected Sets

- Definition: A region  $R \subset S$  is said to be connected under  $c(s)$  if for all  $s, r \in R$  there exists a sequence of  $M$  pixels,  $s_1, \dots, s_M$  such that

$$s_1 \in c(s), s_2 \in c(s_1), \dots, s_M \in c(s_{M-1}), r \in c(s_M)$$

i.e. there is a connected path from  $s$  to  $r$ .

## Example of Connect Sets

- Consider the following image  $X_s$

$$\begin{array}{ccccccc}
 1 & 1 & 1 & 0 & 0 & 0 \\
 1 & 1 & 1 & 0 & 0 & 0 \\
 1 & 1 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 1 & 1 \\
 0 & 0 & 0 & 1 & 1 & 1 \\
 0 & 0 & 0 & 1 & 1 & 1
 \end{array}
 \quad
 \begin{aligned}
 S_1 &= \{s : X_s = 1\} \\
 S_0 &= \{s : X_s = 0\}
 \end{aligned}$$

- Define  $c(s) = \{r \in \partial s : X_r = X_s\}$
- Result
  - 4-point neighborhood  $\Rightarrow S_0$  and  $S_1$  are not connected sets
  - 8-point neighborhood  $\Rightarrow S_0$  and  $S_1$  **are** connected sets!

## Region Growing

- Idea - Find a connected set by growing a region from a seed point  $s_0$
- Assume that  $c(s)$  is given

$ClassLabel = 1$

Initialize  $Y_r = 0$  for all  $r \in S$

ConnectedSet( $s_0, Y, ClassLabel$ ) {

$B \leftarrow \{s_0\}$

While  $B$  is not empty {

$s \leftarrow$  any element of  $B$

$B \leftarrow B - \{s\}$

$Y_s \leftarrow ClassLabel$

$B \leftarrow B \cup \{r : r \in c(s) \text{ and } Y_r = 0\}$

}

return(Y)

}

# Region Growing Example (1)

The list of $(i, j) \in B$		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	1	

The segmentation $Y$							
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	0	0	0	0	0	0
	2	0	0	0	0	0	0
	3	0	0	0	0	0	0
	4	0	0	0	0	0	0

## Region Growing Example (2)

The list of  
 $(i, j) \in B$   
 $(1,0)$

		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
2	0	1	1	0	0	0	0
3	0	1	1	0	0	0	0
4	0	1	0	0	1	0	0

The segmentation  $Y$

		The segmentation $Y$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0

## Region Growing Example (3)

The list of  
 $(i, j) \in B$   
 $(1,1)$

		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
2	0	1	1	0	0	0	0
3	0	1	1	0	0	0	0
4	0	1	0	0	1	0	0

The segmentation  $Y$

		The segmentation $Y$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0

## Region Growing Example (4)

The list of $(i, j) \in B$		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	1	

The segmentation $Y$							
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	0	0	0	0
	3	0	0	0	0	0	0
	4	0	0	0	0	0	0

## Region Growing Example (5)

The list of $(i, j) \in B$		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	1	

The segmentation $Y$							
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	0	0	0	0
	4	0	0	0	0	0	0

## Region Growing Example (6)

The list of $(i, j) \in B$		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	1	

The segmentation $Y$							
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	0	

## Region Growing Example (7)

The list of $(i, j) \in B$		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	1	

The segmentation $Y$							
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	0	

## Region Growing Example (8)

The list of $(i, j) \in B$		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	1	

The segmentation $Y$							
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
	2	0	1	1	0	0	0
	3	0	1	1	0	0	0
	4	0	1	0	0	0	

## Region Growing Example (9)

The list of  
 $(i, j) \in B$   
 empty

		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
2	0	1	1	0	0	0	0
3	0	1	1	0	0	0	0
4	0	1	0	0	1	0	0

The segmentation  $Y$

		The segmentation $Y$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
2	0	1	1	0	0	0	0
3	0	1	1	0	0	0	0
4	0	1	0	0	0	0	0

## Connected Components Extraction

- Iterate through each pixel in the image.
- Extract connected set for each unlabeled pixel.

$ClassLabel = 1$

Initialize  $Y_r = 0$  for  $r \in S$

For each  $s \in S$  {

    if( $Y_s = 0$ ) {

        ConnectedSet( $s, Y, ClassLabel$ )

$ClassLabel \leftarrow ClassLabel + 1$

    }

}

# Connected Components Extraction Example (1)

$s = (i, j);$	The image $X$				
$ClassLabel$					
$(0, 0); 1$					
	j	0	1	2	3
i	0	1	0	0	0
	1	1	1	0	0
	2	0	1	1	0
	3	0	1	1	0
	4	0	1	0	0

The segmentation  $Y$ 

	j	0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

## Connected Components Extraction Example (2)

$s = (i, j);$   
 $ClassLabel$   
 $(0, 1); 2$

		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
2	0	1	1	0	0	0	0
3	0	1	1	0	0	0	0
4	0	1	0	0	1	0	0

The segmentation  $Y$

		The segmentation $Y$					
		j	0	1	2	3	4
i	0	1	2	2	2	2	2
	1	1	1	2	2	2	2
2	0	1	1	2	2	2	2
3	0	1	1	2	2	2	2
4	0	1	2	2	0	0	0

## Connected Components Extraction Example (3)

$s = (i, j);$   
 $ClassLabel$   
 $(2, 0); 3$

		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
2	0	1	1	0	0	0	0
3	0	1	1	0	0	0	0
4	0	1	0	0	1	0	0

The segmentation  $Y$

		The segmentation $Y$					
		j	0	1	2	3	4
i	0	1	2	2	2	2	2
	1	1	1	2	2	2	2
2	3	1	1	2	2	2	2
3	3	1	1	2	2	2	2
4	3	1	2	2	0	0	0

## Connected Components Extraction Example (4)

$s = (i, j);$   
 $ClassLabel$   
 $(4, 4); 4$

		The image $X$					
		j	0	1	2	3	4
i	0	1	0	0	0	0	0
	1	1	1	0	0	0	0
2	0	1	1	0	0	0	0
3	0	1	1	0	0	0	0
4	0	1	0	0	1	0	0

The segmentation  $Y$

		The segmentation $Y$					
		j	0	1	2	3	4
i	0	1	2	2	2	2	2
	1	1	1	2	2	2	2
2	3	1	1	2	2	2	2
3	3	1	1	2	2	2	2
4	3	1	2	2	4	2	2