EE 637 Final Exam

May 1, Spring 2002

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Instructions:

- Follow all instructions carefully!
- This is a 120 minute exam containing **four** problems.
- You may **only** use your brain and a pencil (or pen) to complete this exam. You **may not** use your book, notes or a calculator.

Good Luck.

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Problem 1.(25pt)

Consider the 2-D error diffusion algorithm specified by the equations

$$
\begin{aligned}
z(m,n) &= Q\left[y(m,n)\right] \\
e(m,n) &= y(m,n) - z(m,n) \\
y(m,n) &= x(m,n) + h(m,n) * e(m,n)
\end{aligned}
$$

where $x(m,n)$ is the input, $z(m,n)$ is the output, and $Q(\cdot)$ is a quantizer with the form

$$
Q[y] = \begin{cases} 1 & \text{if } Y > 0.5 \\ 0 & \text{if } Y \leq 0.5 \end{cases}
$$

where m is the column and n is the row.

a) For this part, assume that

$$h(m,n) = \delta(m-1,n)$$

$$e(m,n) = 0 \text{ for } m < 0 \text{ or } n < 0$$

and $x(m,n)$ is given by

$x(m,n)$	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$
$n=0$	0.25	0.25	0.25	0.25	0.25
$n=1$	0.25	0.25	0.25	0.25	0.25
$n=2$	0.25	0.25	0.25	0.25	0.25

Then compute the modified input $y(m,n)$

$y(m,n)$	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$
$n=0$	0.25	0.5	0.75	0	0.25
$n=1$	0.25	0.5	0.75	0	0.25
$n=2$	0.25	0.5	0.75	0	0.25

and compute the output $z(m,n)$.

$z(m,n)$	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$
$n=0$	0	0	1	0	0
$n=1$	0	0	1	0	0
$n=2$	0	0	1	0	0

b) For this part, assume that

$$h(m,n) = \delta(m-1, n-1)$$

$$e(m,n) = 0 \text{ for } m < 0 \text{ or } n < 0$$

and $x(m,n)$ is given by

$x(m,n)$	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$
$n=0$	0.25	0.25	0.25	0.25	0.25
$n=1$	0.25	0.25	0.25	0.25	0.25
$n=2$	0.25	0.25	0.25	0.25	0.25

Then compute the modified input $y(m,n)$

$y(m,n)$	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$
$n=0$	0.25	0.25	0.25	0.25	0.25
$n=1$	0.25	0.5	0.5	0.5	0.5
$n=2$	0.25	0.5	0.75	0.75	0.75

and compute the output $z(m,n)$.

$z(m,n)$	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$
$n=0$	0	0	0	0	0
$n=1$	0	0	0	0	0
$n=2$	0	0	1	1	1

c) Calculate an expression for the display error $d(m,n) = z(m,n) - x(m,n)$ in terms of the quantizer error $e(m,n)$ and the filter $h(m,n)$.

d) Calculate an expression for the DSFT of the display error $D(e^{j\mu}, e^{j\nu}) = Z(e^{j\mu}, e^{j\nu}) - X(e^{j\mu}, e^{j\nu})$ in terms of the DSFT of the quantizer error $E(e^{j\mu}, e^{j\nu})$ and the filter $H(e^{j\mu}, e^{j\nu})$.

c)

$$\begin{aligned} d(m,n) &= z(m,n) - x(m,n) \\ &= y(m,n) - e(m,n) - x(m,n) \\ &= y(m,n) - x(m,n) - e(m,n) \\ &= h(m,n) * e(m,n) - e(m,n) \\ &= -\big(\delta(m,n) - h(m,n)\big) * e(m,n) \end{aligned}$$

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d) $D(e^{j\mu}, e^{j\nu}) = DSFT\{d(m,n)\}$

$= DSFT\left\{ -\left(\delta(m,n) - h(m,n)\right) * e(m,n) \right\}$

$= -\left(1 - H(e^{j\mu}, e^{j\nu})\right) E(e^{j\mu}, e^{j\nu})$

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Problem 2.(25pt)

Consider the TV signal $r(t)$ formed by scanning the entire scene f(x,y,t) with the form

$$f(x, y, t) = g(x)$$

where x is the horizontal coordinate (increasing to the right) that ranges over $0 < x < 1$, and y is the vertical coordinate (increasing toward the bottom) that ranges over $0 < y < 1$. The signal $r(t)$ is obtained using interlaced scanning with a total of 525 lines per frame and 30 frames per second. Assume that the time between the end of one scan line and the beginning of the next is zero.

a) How many fields are scanned per second?

b) How many lines are scanned per second?

c) Write an expression for $r(t)$.

d) Write and expression for $R(f)$ the CTFT of $r(t)$.

e) Sketch the form of the magnitude of a typical spectrum $|R(f)|$.

a) 60 fields per second

b) 525 (30) lines per second

$= 15{,}750$ lps

c) The first line is $g((15{,}750)t)$.

Windowing a single line results in

$$g(t/T)\,\mathrm{rect}\left(\frac{t - T/2}{T}\right)$$

Where $T = 1/15750$ sec

So repeating with period T results in

$$r(t) = \mathrm{rep}_T\left\{ g(t/T)\,\mathrm{rect}\left(\frac{t - T/2}{T}\right)\right\}$$

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d) Let $G(f) = CTFT\left\{ g(x)\, rect(x - \frac{1}{2}) \right\}$

Then

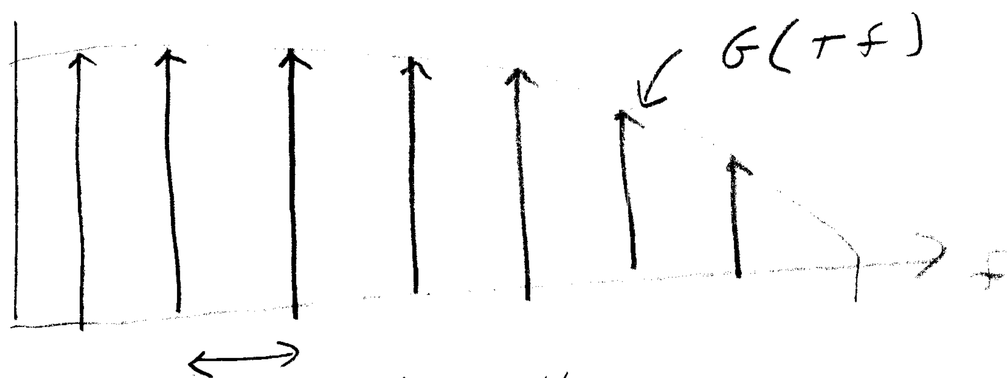
$$R(f) = \frac{1}{T} comb_{1/T}\left\{ T G(Tf) \right\}$$

$$= comb_{1/T}\left\{ G(Tf) \right\}$$

$$= G(Tf) \sum_{k=-\infty}^{\infty} \delta(f - k/T)$$

$$|R(f)| = |G(Tf)| \sum_{k=-\infty}^{\infty} \delta(f - k/T)$$

e)



$\frac{1}{T} = \frac{1}{15750}$ Hz

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Problem 3.(25pt)

Let the image $y(m,n)$ be formed by applying 2-D interpolation by a factor of $L = 2$ to the signal $x(m,n)$ with an interpolation filter of the form

$$\begin{aligned} h(m,n) &= 0.25\delta(m-1,n-1) + 0.5\delta(m,n-1) + 0.25\delta(m+1,n-1) \\ &\quad +0.5\delta(m-1,n) + \delta(m,n) + 0.5\delta(m+1,n) \\ &\quad +0.25\delta(m-1,n+1) + 0.5\delta(m,n+1) + 0.25\delta(m+1,n+1) \end{aligned}$$

a) Use a free boundary condition to compute $y(m,n)$ for the input $x(m,n)$ given by

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{matrix}$$

b) Compute $H(e^{j\mu}, e^{j\nu})$ the DSFT of the filter $h(m,n)$.

c) Write an expression for $Y(e^{j\mu}, e^{j\nu})$ in terms of $X(e^{j\mu}, e^{j\nu})$ and $H(e^{j\mu}, e^{j\nu})$.

d) What are the advantages and disadvantages of this interpolation method?

a) $$x(m,n) \to \boxed{\uparrow 2} \xrightarrow{z(m,n)} \boxed{H} \to y(m,n)$$

$z(m,n)$

$$\begin{matrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$y(m,n)$

$$\begin{matrix} 0 & 1/2 & 1 & 1 & 1 \\ 0 & 1/2 & 1 & 1 & 1 \\ 0 & 1/2 & 1 & 1 & 1 \\ 0 & 1/4 & 1/2 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

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b) $h(m,n) = f(m)\,f(n)$

where

$$f(m) = 0.5\,\delta(m-1) + \delta(m) + 0.5\,\delta(m+1)$$

$$F(e^{j\omega}) = DTFT\{f(m)\}$$

$$= 1 + 0.5\,e^{j\omega} + 0.5\,e^{-j\omega}$$

$$= 1 + \frac{e^{j\omega} + e^{-j\omega}}{2} = 1 + \cos(\omega)$$

$$H(e^{j\mu}, e^{j\nu}) = DSFT\{h(m,n)\}$$

$$= (1+\cos(\mu))(1+\cos(\nu))$$

c) $Y(e^{j\mu}, e^{j\nu})$

$$= \sum_{k=0}^{1}\sum_{l=0}^{1} H\left(e^{j(\mu-2\pi k)/2}, e^{j(\nu-2\pi l)/2}\right) \cdot X\left(e^{j(\mu-2\pi k)/2}, e^{j(\nu-2\pi l)/2}\right)$$

d) Advantages:

Easy to compute

Reduces aliasing compared to pixel replication

Disadvantages:

Softens image due to attenuation in pass band

Allows some aliased frequency energy.

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Problem 4.(25pt)

Consider the set of data $\{x_n\}_{n=0}^{N-1}$. We would like to estimate a "central value" using a method known as M-estimation. To do this we compute the following function

$$\begin{aligned} y &= f(x) \\ &= \arg\min_{\theta} \left\{ \sum_{n=0}^{N-1} \rho(x_n - \theta) \right\} \end{aligned}$$

where ρ is a function with the properties that $\rho(\Delta) \geq 0$ and $\rho(-\Delta) = \rho(\Delta)$.

a) What is the value of y when

$$\rho(\Delta) = \Delta^2$$

b) ~~a~~) What is the value of y when

$$\rho(\Delta) = |\Delta|$$

c) Derive an expression for computing y when

$$\rho(\Delta) = |\Delta|^{0.5}$$

d) When $\rho(\Delta) = |\Delta|^{0.5}$, is the function $y = f(x)$ linear? Is it homogeneous? Justify your answers.

a) $y = \arg\min_{\theta} \sum_{n=0}^{N-1} (x_n - \theta)^2$

$$\frac{d}{d\theta} \sum_{n=0}^{N-1} (x_n - \theta)^2 = 0$$

$$\sum_{n=0}^{N-1} (x_n - \theta)(-\theta) = 0$$

$$\theta \sum_{n=0}^{N-1} x_n = \theta^2 N$$

$$\frac{1}{N} \sum_{n=0}^{N-1} x_n = \theta$$

so $y = \frac{1}{N} \sum_{n=0}^{N-1} x_n \Leftarrow$ mean value

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b) $$\frac{d}{d\theta} \sum_{n=0}^{N-1} |x_n - \theta| = 0$$

$$\sum_{n=0}^{N-1} \text{sign}(x_n - \theta)(-1) = 0$$

$$\sum_{n=0}^{N-1} \text{sign}(X_n - \theta) = 0$$

$$y = \text{median}\{X_0, \dots, X_{N-1}\}$$

c) $$\frac{d}{d\theta} \sum_{n=0}^{N-1} (x_n - \theta)^{0.5} = 0$$

$$\sum_{n=0}^{N-1} 0.5\,|x_n - \theta|^{-0.5} \text{sign}(\theta - x_n) = 0$$

$$\sum_{n=0}^{N-1} (x_n - \theta)^{-0.5} \text{sign}(x_n - \theta) = 0$$

$$y = \text{root}_\theta \left\{ \sum_{n=0}^{N-1} |x_n - \theta|^{-0.5} \text{sign}(x_n - \theta) \right\}$$

d) <u>Linear: No</u>

Let $N = 3$ and $(x_1^1, x_2^1, x_3^1) = (0, 1, 0)$

then $y^1 = 0$

Let $N = 3$ and $(x_1^2\ x_2^2\ x_3^2) = (0, 0, 1)$

then $y^2 = 0$

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but notice that

$$(x_1^3, x_2^3, x_3^3) = (x_1^1, x_2^1, x_3^1) + (x_1^2, x_2^2, x_3^2)$$

$$= (0, 1, 1)$$

$$y^3 = 1 \neq y^1 + y^2 = 0 + 0 = 0$$

<u>Homogeneous: Yes</u>

Let $(x_1', x_2', x_3') = \alpha(x_1, x_2, x_3)$

for $\alpha > 0$

$$y' = \arg\min_\theta \sum_{n=0}^{N-1} (x_n' - \theta)^2$$

$$= \arg\min_\theta \sum_{n=0}^{N-1} (\alpha x_n - \theta)^2$$

$$= \arg\min_\theta \sum_{n=0}^{N-1} \alpha^2 (x_n - \theta/\alpha)^2$$

$$= \arg\min_\theta \sum_{n=0}^{N-1} (x_n - \theta/\alpha)^2$$

$$= \alpha \arg\min_\theta \sum_{n=0}^{N-1} (x_n - \theta)^2$$

$$= \alpha y$$