EE 637 Midterm Exam #1

February 20, Spring 2002

Name: ______________________________

Instructions:

- Follow all instructions carefully!
- This is a 50 minute exam containing **three** problems.
- You may **only** use your brain and a pencil (or pen) to complete this exam. You **may not** use your book, notes or a calculator.
- This is also homework #1. Please keep it. Complete it as a homework, and hand it in by Friday at 5:00PM.

Good Luck.

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Problem 1.(25pt)

Consider the DT random process $X(n)$ formed by i. i. d. random variables with

$$P\{X(n) = 1\} = P\{X(n) = -1\} = 0.5$$

Also consider the DT random process

$$Y(n) = \sum_{k=-\infty}^{\infty} X(k)p(n-k)$$

where $p(n)$ is a real valued DT function with a DTFT of $P(e^{j\omega})$.

a) Is $Y(n)$ a stationary random process? Justify your answer.

b) Compute $R_x(k)$ autocorrelation and $S_x(e^{j\omega})$ the powerspectrum for $X(n)$.

c) Compute $R_y(k)$ autocorrelation and $S_y(e^{j\omega})$ the powerspectrum for $Y(n)$.

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Problem 2.(50pt)

The forward projection is defined by the operation

$$\begin{aligned} p_\theta(r) &= \mathcal{FP}\{f(x,y)\} \\ &= \int_{-\infty}^{\infty} f(r\cos(\theta) - z\sin(\theta), r\sin(\theta) + z\cos(\theta))\, dz \end{aligned}$$

The back projection is defined by the operation

$$\begin{aligned} h(x,y) &= \mathcal{BP}\{g_\theta(r)\} \\ &= \int_0^{\pi} g_\theta(x\cos(\theta) + y\sin(\theta))d\theta \end{aligned}$$

Furthermore, define the composition of forward and back projection as the 2-D continuous space system $\mathcal{T}\{\cdot\}$

$$\begin{aligned} h(x,y) &= \mathcal{T}\{f(x,y)\} \\ &= \mathcal{BP}\{\mathcal{FP}\{f(x,y)\}\} \end{aligned}$$

a) Compute

$$p_\theta(r) = \mathcal{FP}\{\delta(x,y)\}$$

the forward projection of the signal $f(x,y) = \delta(x,y)$.

b) Show that

$$\frac{1}{\sqrt{x^2+y^2}} = \mathcal{T}\{\delta(x,y)\} \ .$$

(Hint: Use the transformation of variables $x = r\cos\phi$ and $y = r\sin\phi$.)

c) Determine

$$w(x,y) = \mathcal{T}\{\delta(x-x_0, y-y_0)\} \ .$$

(Hint: You may want to make a graphical argument.)

d) Is the system $\mathcal{T}\{\cdot\}$ linear? Justify your answer.

e) Is the system $\mathcal{T}\{\cdot\}$ space invariant? Justify your answer.

f) Propose a method for inverting the system $\mathcal{T}\{\cdot\}$.

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Problem 3.(25pt)

Consider an MRI system for which

$$\omega = LM$$

where ω is the resonance frequency in radians per second, L is the Lamar constant and M is the magnetic field; and let $s(t)$ denote the RF signal used to excite the target.

Furthermore, assume that the magnetic field can be controlled via three gradient coils, so that the magnetic field is given by

$$M(x, y, z) = M_0 + xG_x + yG_y + zG_z$$

where (x, y, z) are the spatial coordinates in 3-D.

a) (Slice select step.) Initially, the gradients are set to $G_x = 0$, $G_y = 0$, and $G_z = G_0 \neq 0$, and target is excited with the RF pulse

$$s(t) = e^{jLM_0 t}\text{sinc}(t) \ .$$

For which values of (x, y, z) do the target atoms resonate? Justify your result.

b) Next the values of the x and y gradients are changed, but the z gradient remains fixed, so that $G_x = G_x(t)$, $G_y = G_y(t)$, and $G_z = G_0 \neq 0$. Let $\phi(t, x, y)$ be the phase of the signal radiated from a voxel of size (dx, dy, dz) at location (x, y), $z = 0$, and time t.

Compute an expression for $\phi(t, x, y)$ assuming that $\phi(0, x, y) = 0$.

c) Let $A(x, y)$ be the magnitude of the signal radiated from the voxel of size (dx, dy, dz) at location (x, y) and $z = 0$. Compute the total radiated signal from a slice of thickness dz at position $z = 0$.

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