

6.

IMAGE CODING (Source Coding)

- Lossless Coding
 - Entropy coding
 - Huffman code
- Lossy Coding
 - JPEG
 - Bit rate depends on quality

7.

Entropy Coding (Lossless)

Let X_n be a i.i.d sequence of Random Variables taking on values in $\{0, \dots, M-1\}$

$$P\{X_n = i\} = p_i \quad 0 \leq i \leq M-1$$

Entropy

$$H(X_n) \triangleq - \sum_{i=0}^{M-1} p_i \log_2 p_i$$

$$= -E[\log_2 p_i]$$

9.

Definitions

Fixed length Code -
code in which σ_i have
same length for all i .

Variable length Code -
code in which σ_i have
different lengths for
different i 's.

8.

How do you code RVIS
into a binary sequence?

Definition: A code is a
mapping from the set
 $\{0, \dots, M-1\}$ to finite binary
sequences.

$M=4$	$0 \rightarrow 01$	$= \sigma_0$
	$1 \rightarrow 10$	$= \sigma_1$
	$2 \rightarrow 0$	$= \sigma_2$
	$3 \rightarrow 10010$	$= \sigma_3$

9.

Problem

Decode: 010

$$\text{CASE 1: } \begin{array}{r|l} 0 & 10 \\ \hline 2 & 1 \end{array}$$

$$\text{CASE 2: } \begin{array}{r|l} 01 & 0 \\ \hline 0 & 2 \end{array}$$

Not Uniquely decodable

10.

Definitions

- A code is Uniquely Decodable if there exist only a single unique decoding.
- A Prefix Code is a specific type of uniquely decodable code in which no code is a prefix of another code.

11.

Theorem: Let C be a uniquely decodable code for the values X_n . Then the ~~average~~ mean code length $\geq H(X_n)$.

$$\bar{n} \triangleq E[|\sigma_{X_n}|] = \sum_{i=0}^{M-1} |\sigma_i| p_i$$

$$\geq H(X_n)$$

Remember:

X_n are i.i.d
 $|\sigma_i| = \text{length of code}$

12.

Question

Can we achieve this bound?

Is the bound tight?

Yes!

Constructive proof

Huffman code

13.

Comments about Huffman Codes

- 1) Huffman codes are variable length prefix codes.
- 2) Uniquely decodable
- 3) $\emptyset$ Low prob. $\Rightarrow$ long code
high prob. $\Rightarrow$ short code

14.

Theorem 1) For a Huffman code, $\bar{n}$ has the property that

$$H(X_n) \leq \bar{n} < H(X_n) + 1$$

- Huffman code is within 1 bit of optimal efficiency.
- Can we do better?

15.

Coding in blocks

• Code blocks of the sequence x_n

$\dots, x_0, \dots, x_{m-1}, x_m, \dots, x_{2m-1}, \dots$

$\underbrace{\hspace{10em}}_{Y_0} \quad \underbrace{\hspace{10em}}_{Y_1}$

$$Y_n = [x_{nm}, \dots, x_{n+m-1}]$$

$$Y_n \in \{0, \dots, M^m - 1\}$$

16.

• It is easily shown that

$$H(Y_n) = m H(X_n)$$

• By theorem 1):

$$(*) \quad H(Y_n) \leq \bar{n}_Y < H(Y_n) + 1$$

$\bar{n}_Y$ = mean # of bits per symbol
for Y_n .

$\bar{n}_X$ = mean # of bits per symbol
for X_n

$$(\bar{n}_X = \bar{n}_Y / m)$$

17.

• Dividing (*) by n results in

$$\frac{1}{n} H(Y_n) \leq \frac{\bar{n}_Y}{n} < \frac{H(Y_n)}{n} + \frac{1}{n}$$

$$H(X_n) \leq \bar{n}_X < H(X_n) + \frac{1}{n}$$

As $n \rightarrow \infty$,

$$\bar{n}_X \approx H(X_n)$$

18.

Comments

1) As the block size m goes to infinity, the bitrate approaches the entropy per input symbol.

$$\lim_{m \rightarrow \infty} \bar{R}_x = H(X_n)$$

2) This assumes that X_n are i.i.d., but a similar analysis can be applied to stationary and ergodic sources.

Algorithm for Constructing a Huffman Code

- Step 1: Search the list of probabilities for the two levels with the lowest probabilities. In our example, these are levels 7 and 8 with probabilities 0.04 and 0, respectively.
 - Step 2: Add the two lowest probabilities together. Form a new list that includes all the probabilities not affected in Step 1, plus the sum generated there. The new list has one less element.
 - Step 3: Go back to Step 1 and repeat the procedure until the list contains only one element.
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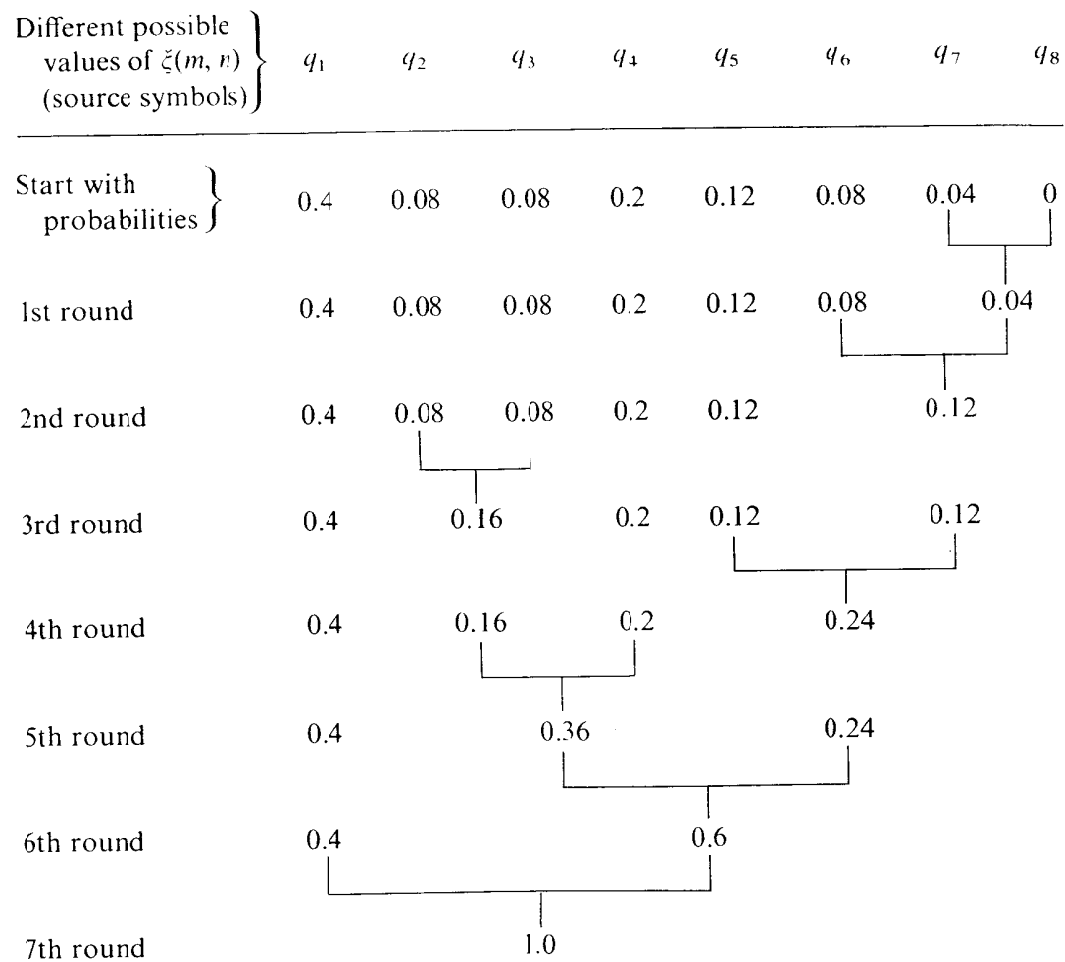


Fig. 25 Building the tree for the Huffman code.

