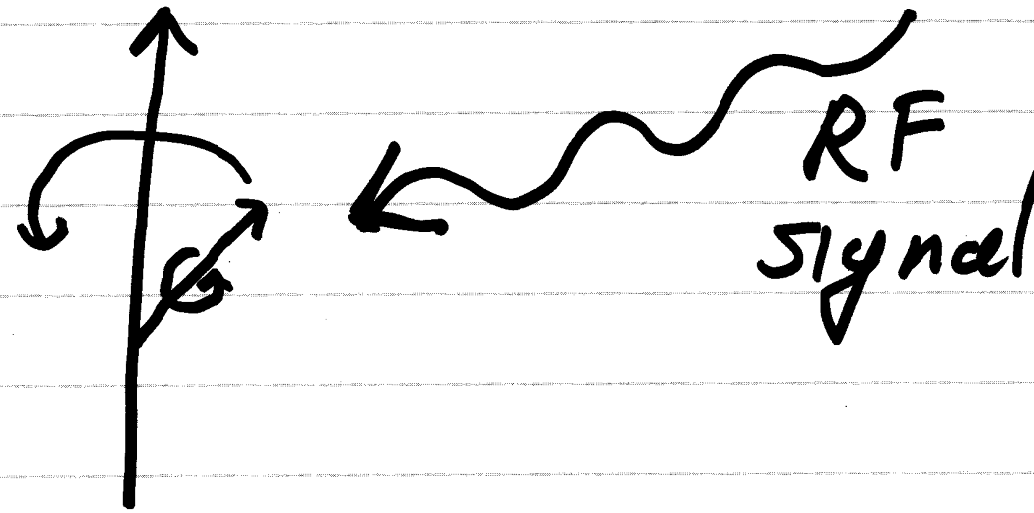
Magnetic Resonance Imaging

- Based on magnetic resonance effect in hydrogen
- Does not require any ionizing radiation
- Numerous modality
  - Functional MRI
  - MRI Tagging
  - high speed MRI

Magnetic Resonance

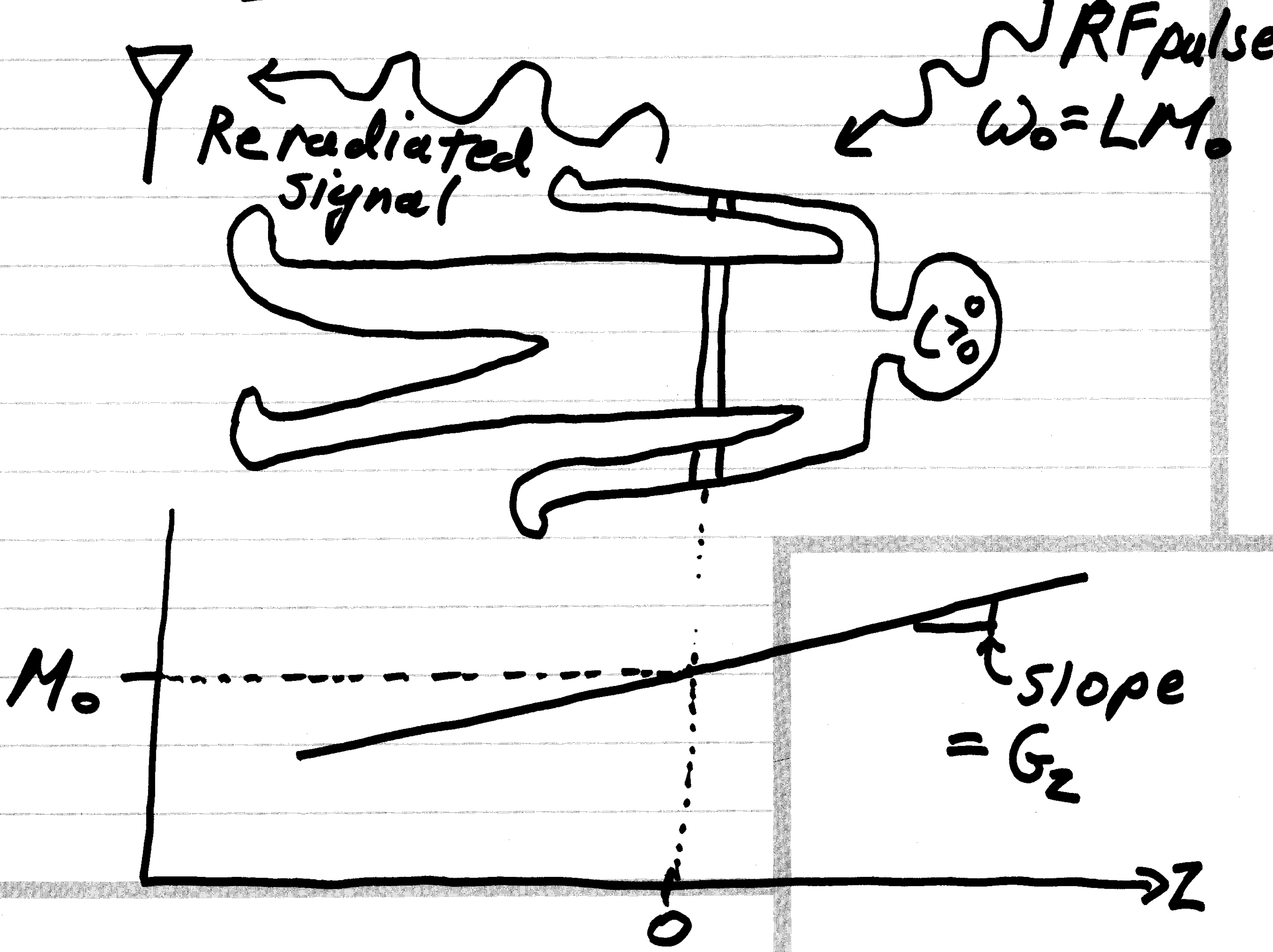


- Atom will process at frequency

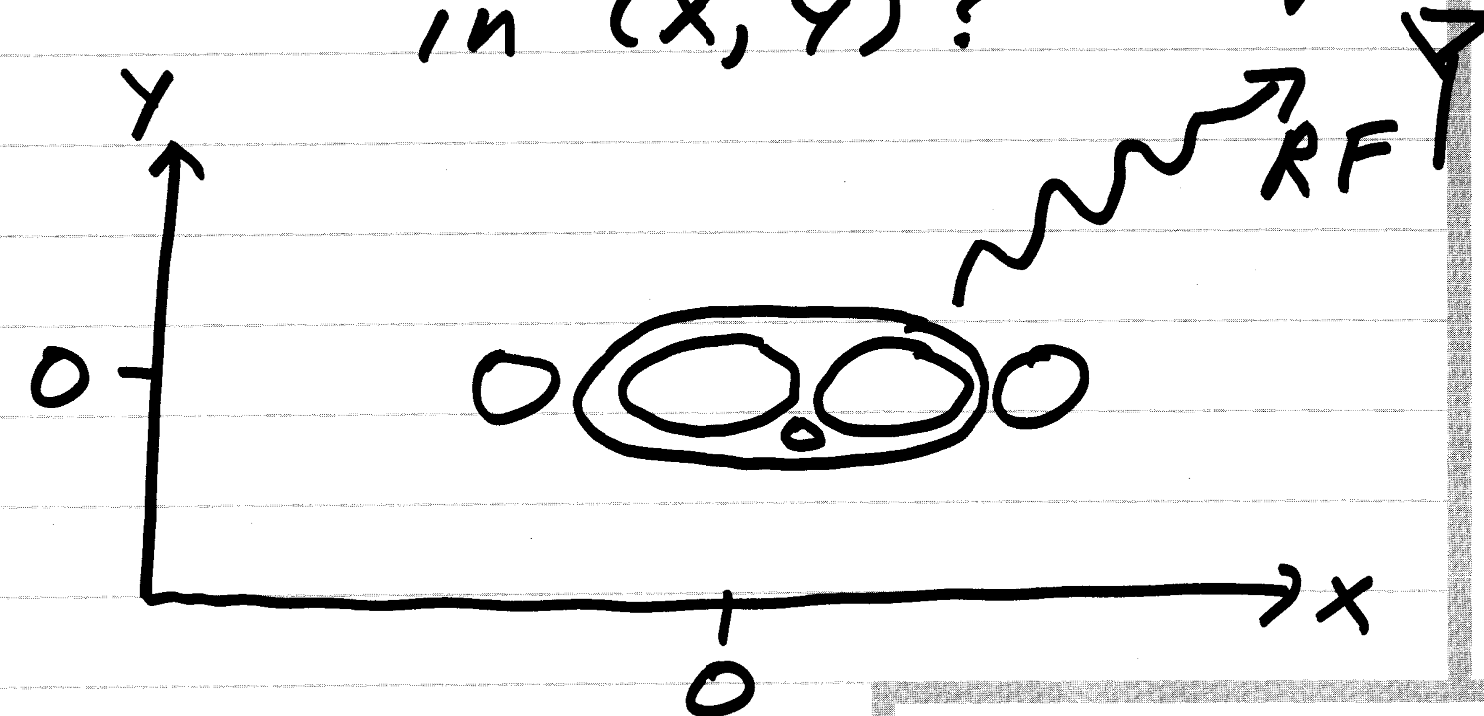
$$\omega_0 = L M_0$$

Laman Frequency

Slice Select



How Do We Form an Image in (X, Y)?



- Vary magnetic field Gradients in X and Y <u>after</u> slice select.

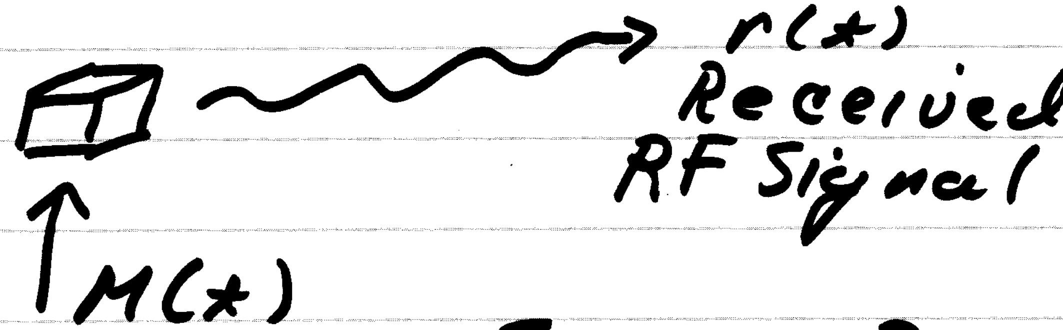
$G_x(t) \Leftarrow$ Gradient in X direction

$G_y(t) \Leftarrow$ Gradient in Y direction

- Total magnetic field

$$M(t) = M_0 + X\,G_x(t) + Y\,G_y(t)$$

Signal Radiated from a Single Voxel



$$r(t) = f(x,y)\exp\{j\phi(t)\}$$

- $f(x,y)$ is weighting of signal due to material properties.
  - Typically T1, T2, or T2* weighted.

- $\phi(t)$ is phase of signal

$$\phi(t) = \int_0^t \left\{ \mathcal{L} M_0 + \mathcal{L} x G_x(\tau) + \mathcal{L} y G_y(\tau) \right\} d\tau$$

$$= t \underbrace{\mathcal{L} M_0}_{\omega_0} + x \underbrace{\int_0^t \mathcal{L} G_x(\tau)\, d\tau}_{K_x(t)} + y \underbrace{\int_0^t \mathcal{L} G_y(\tau)\, d\tau}_{K_y(t)}$$

Define

K-space
$$\begin{cases} K_x(t) \triangleq \int_0^t \mathcal{L} G_x(\tau)\, d\tau \\ K_y(t) \triangleq \int_0^t \mathcal{L} G_y(\tau)\, d\tau \end{cases}$$

Instantaneous Frequency

$$\text{Instantaneous Freq.} = \frac{d\phi(t)}{dt}$$

$$= L M(t) = \text{Larmor Freq.}$$

$$\phi(t) = \int_0^t L M(\cancel{t}\,\tau)\, d\tau$$

$$M(t) = M_0 + x G_x(t) + y G_y(t)$$

$$\phi(t) = t\omega_0 + x K_x(t) + y K_y(t)$$

$$r(t) = f(x,y)\, e^{j\omega_0 t} \cdot e^{j(x K_x(t) + y K_y(t))}$$

signal radiated from a single voxel

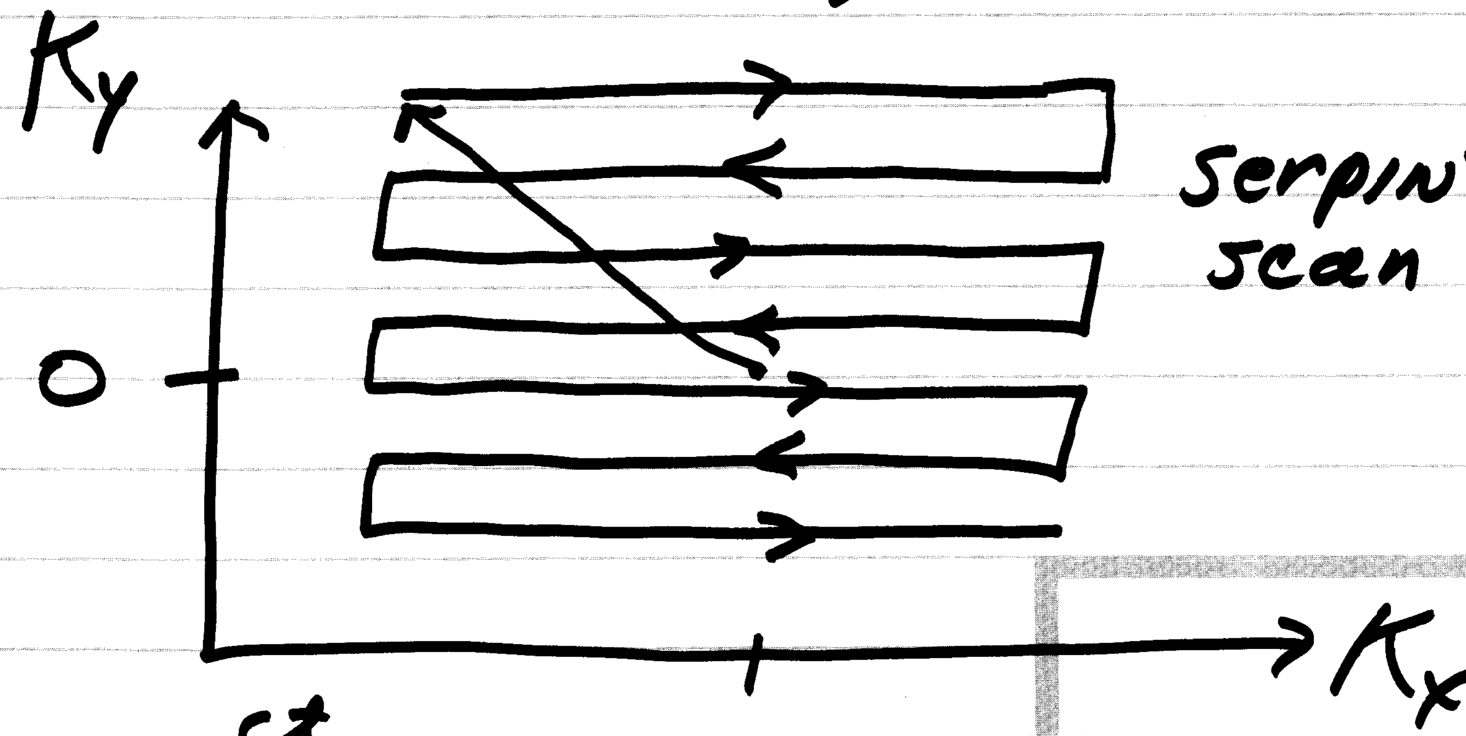
Signal Radiated From Selected Slice

$$R(t) = \iint_{\mathbb{R}\,\mathbb{R}} f(x,y)\, e^{j\omega_0 t} \cdot e^{j(x K_x(t) + y K_y(t))}\, dx\, dy$$

$$= e^{j\omega_0 t} \cdot \int_{\mathbb{R}^2} f(x,y)\, e^{j(x K_x(t) + y K_y(t))}\, dx\, dy$$

(inverse) CSFT!!

Echo Planer Imaging
(EPI)



$$K_x = \int^t \gamma G_x(\tau)\,d\tau$$

$$K_y = \int^t \gamma G_y(\tau)\,d\tau$$

Gradient Wave forms of EPI