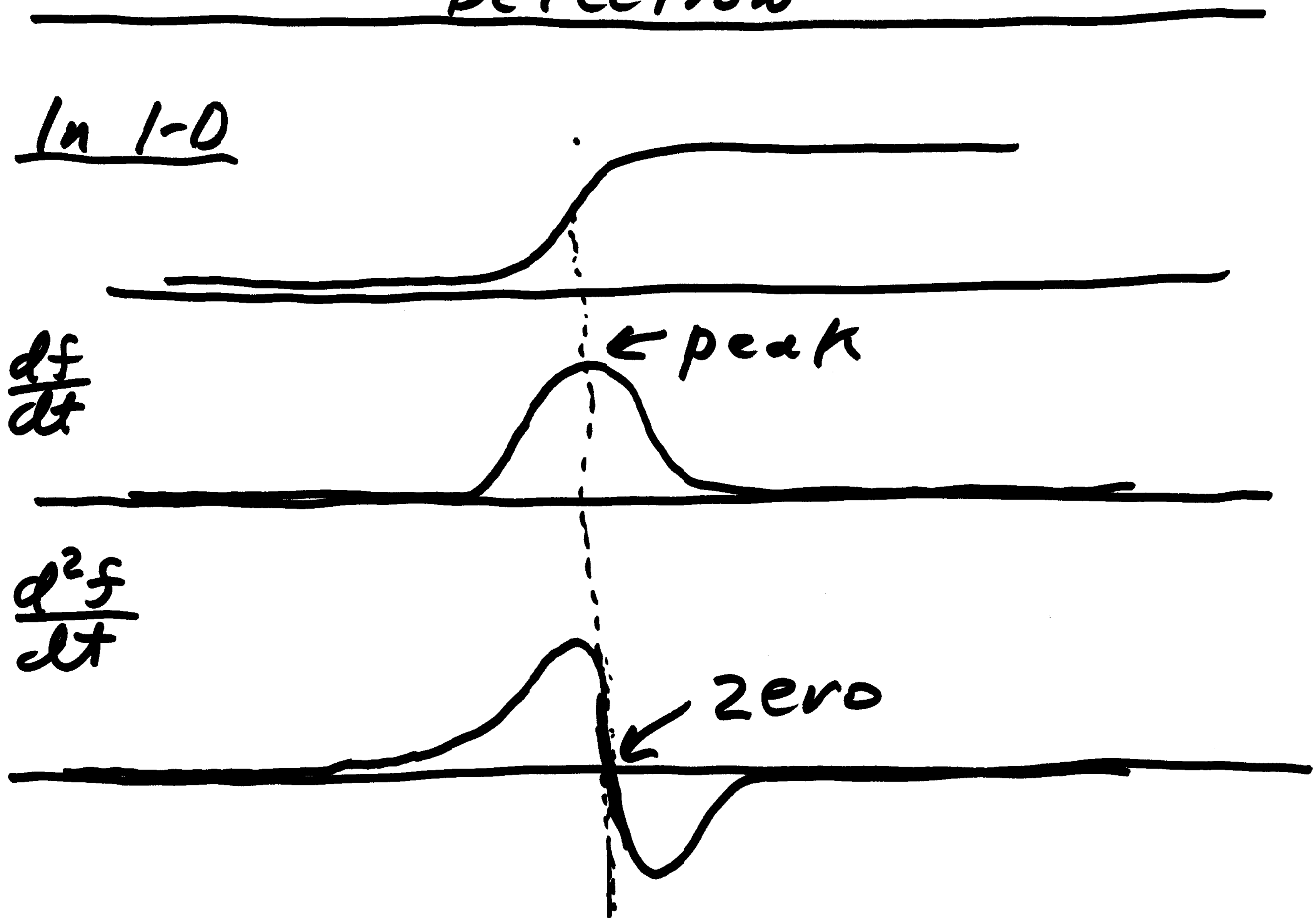
Use of Second Derivative in Edge Detection



Idea

- Look for points such that

$$\frac{d^2 f}{dt^2} = 0$$

- Problem: Flat regions also have zero $\frac{d^2 f}{dx^2}$!

- Condition:

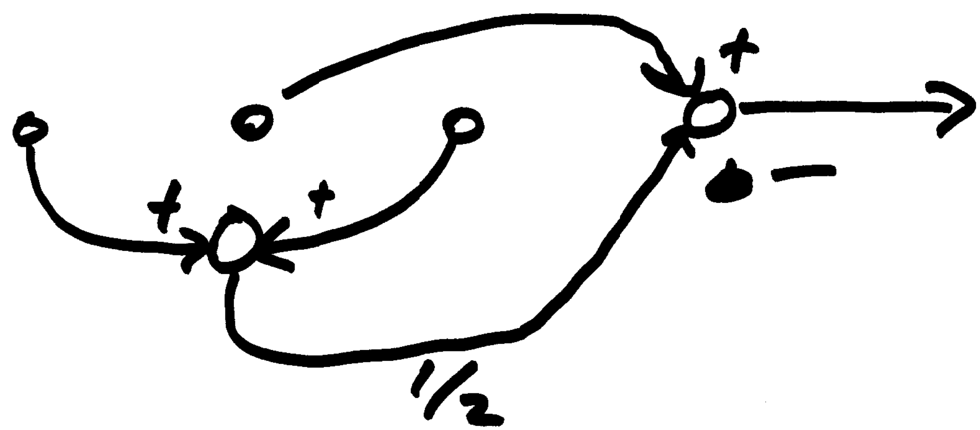
$$\left(\frac{d^2 f}{dt^2} = 0\right) \text{ and } \left(\left|\frac{\partial f}{\partial x}\right| > T\right)$$

Discrete Approximation to Second Derivative

Cont. Time	Disc. Time	Disc. Op.
$\frac{df}{dt}$	$\boxed{-1}\ +1$ on $-1\ \boxed{+1}$	$f(n+1)-f(n)$ on $f(n)-f(n-1)$
$\frac{d^2f}{dt^2}$	$\boxed{-1}\ +1$ $-(-1\ \boxed{+1})$ $=$ $+1\ \ -2\ \ +1$	$-2f(n)$ $+f(n-1)+f(n+1)$ on $-2\left(f(n)-\frac{f(n-1)+f(n+1)}{2}\right)$

<u>In 1-D</u>

$$-\frac{1}{2}\frac{d^2 f}{dx^2} = f(n) - \frac{f(n+1)+f(n-1)}{2}$$



$$-\frac{1}{2} \quad \boxed{1} \quad -\frac{1}{2}$$

In 2-D

$$\nabla^2 f = \nabla \cdot \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$-\frac{1}{2}\nabla^2 f = -\frac{1}{2}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$$

Discrete Approximation

$$\begin{array}{ccc} & -1/2 & \\ -\frac{1}{2} & \boxed{1+1} & -\frac{1}{2} \\ & -1/2 & \end{array}$$

$$-\frac{1}{4}\nabla^2 f = \begin{array}{ccc} & -1/4 & \\ -1/4 & \boxed{1} & -1/4 \\ & -1/4 & \end{array}$$

$$H(e^{i0}, e^{i0}) = 0$$

Using Laplacian in Edge Detection

We would like to find points so that

$$\nabla^2 f(x,y) = 0$$

To detect edges

$$\left.\begin{matrix} \nabla^2 f(x,y) = 0 \\ \text{and} \\ |\nabla f(x,y)| > T \end{matrix}\right\} \Longrightarrow \text{Detect Edge}$$