Space Domain Models for Optical Imaging Systems

- Consider an imaging system with real world image $f(x, y)$, focal plane image $g(x, y)$, and magnification M. Then the behavior of the system may be modeled as:

$$\begin{aligned} g(x,y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi,\eta)h(x-M\xi, y-M\eta)d\xi d\eta \\ &= \frac{1}{M^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left(\frac{\xi}{M},\frac{\eta}{M}\right)h(x-\xi, y-\eta)d\xi d\eta \end{aligned}$$

Define the function

$$\tilde{f}(x,y) \triangleq f\left(\frac{\xi}{M},\frac{\eta}{M}\right)$$

- Then the imaging system act like a 2-D convolution.

$$g(x,y) = \frac{1}{M^2}h(x,y) * \tilde{f}(x,y)$$

Point Spread Functions for Optical Imaging Systems

- Definition: $h(x, y)$ is known as the *point spread function* of the imaging system.

$$g(x, y) = \frac{1}{M^2} h(x, y) * \tilde{f}(x, y)$$

- Notice that when $f(x, y) = \delta(x, y)$

$$\begin{aligned} g(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\xi, \eta) h(x - M\xi, y - M\eta) d\xi d\eta \\ &= h(x, y) \end{aligned}$$

Transfer Functions for Optical Imaging Systems

- In the frequency domain,

$$G(u,v) = \tilde{F}(u,v)\,\frac{1}{M}H(u,v)$$

$$\begin{array}{rcl} g(x,y) & \stackrel{CSFT}{\Leftrightarrow} & G(u,v) \\ h(x,y) & \stackrel{CSFT}{\Leftrightarrow} & H(u,v) \\ \tilde{f}(x,y) & \stackrel{CSFT}{\Leftrightarrow} & \tilde{F}(u,v) \end{array}$$

- The *Optical Transfer Function (OTF)* is

$$\frac{H(u,v)}{H(0,0)}$$

- The *Modulation Transfer Function (MTF)* is

$$\left|\frac{H(u,v)}{H(0,0)}\right|$$