# Connected Component Analaysis

- Once region boundaries have been detected, it is often useful to extract regions which are not separated by a boundary.
- Any set of pixels which is not separated by a boundary is call connected.
- Each maximal region of connected pixels is called a connected component.
- The set of connected components partition an image into segments.
- Image segmentation is an useful operation in many image processing applications.

# Connected Neighbors

- Let $\partial s$ be a neighborhood system.
  - 4-point neighborhood system
  - 8-point neighborhood system
- Let $c(s)$ be the set of neighbors that are connected to the point $s$.

  For all $s$ and $r$, the set $c(s)$ must have the properties that
  - $c(s) \subset \partial s$
  - $r \in c(s) \Leftrightarrow s \in c(r)$
- Example:

$$c(s) = \{r \in \partial s : X_r = X_s\}$$

- Example:

$$c(s) = \{r \in \partial s : |X_r - X_s| < Threshold\}$$

- In general, computation of $c(s)$ might be very difficult, but we won't worry about that now.

# Connected Sets

- Definition: A region $R \subset S$ is said to be connected under $c(s)$ if for all $s, r \in R$ there exists a sequence of $M$ pixels, $s_1, \cdots, s_M$ such that

$$s_1 \in c(s), s_2 \in c(s_1), \cdots, s_M \in c(s_{M-1}), r \in c(s_M)$$

  i.e. there is a connected path from $s$ to $r$.

# Example of Connect Sets

- Consider the following image $X_s$

$$
\begin{array}{cccccc}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{array}
\qquad\qquad
\begin{aligned}
S_1 &= \{s : X_s = 1\} \\
S_0 &= \{s : X_s = 0\}
\end{aligned}
$$

- Define $c(s) = \{r \in \partial s : X_r = X_s\}$
- Result
  - 4-point neighborhood $\Rightarrow$ $S_0$ and $S_1$ are not connected sets
  - 8-point neighborhood $\Rightarrow$ $S_0$ and $S_1$ **are** connected sets!

# Region Growing

- Idea - Find a connected set by growing a region from a seed point $s_0$
- Assume that $c(s)$ is given

$ClassLabel = 1$
Initialize $Y_r = 0$ for all $r \in S$
ConnectedSet$(s_0, Y, ClassLabel)$ {
  $B \leftarrow \{s_0\}$
  While $B$ is not empty {
    $s \leftarrow$ any element of $B$
    $B \leftarrow B - \{s\}$
    $Y_s \leftarrow ClassLabel$
    $B \leftarrow B \cup \{r : r \in c(s)$ and $Y_r = 0\}$
  }
  return(Y)
}

# Region Growing Example (1)

The list of
$(i, j) \in B$
(0,0)

The image $X$

| | j=0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| i 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | j=0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| i 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 |

# Region Growing Example (2)

The list of
$(i, j) \in B$
(1,0)

The image $X$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0 |
| | 4 | 0 | 0 | 0 | 0 | 0 |

# Region Growing Example (3)

The list of

$(i, j) \in B$

(1,1)

The image $X$

| | | j 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0 |
| | 4 | 0 | 0 | 0 | 0 | 0 |

# Region Growing Example (4)

The list of
$(i, j) \in B$
(2,1)

The image $X$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 0 | 0 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0 |
| | 4 | 0 | 0 | 0 | 0 | 0 |

# Region Growing Example (5)

The list of
$(i, j) \in B$
(3,1)
(2,2)

The image $X$

$$
\begin{array}{cc|ccccc}
 & & \text{j} & & & & \\
 & & 0 & 1 & 2 & 3 & 4 \\
\hline
\text{i} & 0 & 1 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 & 0 \\
 & 2 & 0 & 1 & 1 & 0 & 0 \\
 & 3 & 0 & 1 & 1 & 0 & 0 \\
 & 4 & 0 & 1 & 0 & 0 & 1
\end{array}
$$

The segmentation $Y$

$$
\begin{array}{cc|ccccc}
 & & \text{j} & & & & \\
 & & 0 & 1 & 2 & 3 & 4 \\
\hline
\text{i} & 0 & 1 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 & 0 \\
 & 2 & 0 & 1 & 1 & 0 & 0 \\
 & 3 & 0 & 1 & 0 & 0 & 0 \\
 & 4 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

# Region Growing Example (6)

The list of $(i, j) \in B$

(4,1)

(3,2)

(2,2)

The image $X$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 0 |

# Region Growing Example (7)

The list of $(i, j) \in B$

(3,2)

(2,2)

The image $X$

| | | j 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 0 |

# Region Growing Example (8)

The list of
$(i, j) \in B$
(2,2)

The image $X$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 0 |

# Region Growing Example (9)

The list of
$(i,j) \in B$
empty

The image $X$

$$
\begin{array}{cc|ccccc}
 & & \text{j} & & & & \\
 & & 0 & 1 & 2 & 3 & 4 \\
\hline
\text{i} & 0 & 1 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 & 0 \\
 & 2 & 0 & 1 & 1 & 0 & 0 \\
 & 3 & 0 & 1 & 1 & 0 & 0 \\
 & 4 & 0 & 1 & 0 & 0 & 1
\end{array}
$$

The segmentation $Y$

$$
\begin{array}{cc|ccccc}
 & & \text{j} & & & & \\
 & & 0 & 1 & 2 & 3 & 4 \\
\hline
\text{i} & 0 & 1 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 & 0 \\
 & 2 & 0 & 1 & 1 & 0 & 0 \\
 & 3 & 0 & 1 & 1 & 0 & 0 \\
 & 4 & 0 & 1 & 0 & 0 & 0
\end{array}
$$

# Connected Components Extraction

- Iterate through each pixel in the image.
- Extract connected set for each unlabeled pixel.

$ClassLabel = 1$

Initialize $Y_r = 0$ for $r \in S$

For each $s \in S$ {

  if($Y_s = 0$) {

    ConnectedSet($s, Y, ClassLabel$)

    $ClassLabel \leftarrow ClassLabel + 1$

  }

}

# Connected Components Extraction Example (1)

$s = (i, j);$
$ClassLabel$
$(0, 0);\ 1$

The image $X$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 0 |

# Connected Components Extraction Example (2)

$s = (i, j)$;
$ClassLabel$
$(0, 1)$; 2

The image $X$

| | j 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| i 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | j 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| i 0 | 1 | 2 | 2 | 2 | 2 |
| 1 | 1 | 1 | 2 | 2 | 2 |
| 2 | 0 | 1 | 1 | 2 | 2 |
| 3 | 0 | 1 | 1 | 2 | 2 |
| 4 | 0 | 1 | 2 | 2 | 0 |

# Connected Components Extraction Example (3)

$s = (i, j);$
$ClassLabel$
$(2, 0); 3$

The image $X$

| | | j 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| i | 0 | 1 | 2 | 2 | 2 | 2 |
| | 1 | 1 | 1 | 2 | 2 | 2 |
| | 2 | 3 | 1 | 1 | 2 | 2 |
| | 3 | 3 | 1 | 1 | 2 | 2 |
| | 4 | 3 | 1 | 2 | 2 | 0 |

# Connected Components Extraction Example (4)

$s = (i, j);$
$ClassLabel$
$(4, 4);\ 4$

The image $X$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 0 | 0 |
| | 4 | 0 | 1 | 0 | 0 | 1 |

The segmentation $Y$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 |
| i | 0 | 1 | 2 | 2 | 2 | 2 |
| | 1 | 1 | 1 | 2 | 2 | 2 |
| | 2 | 3 | 1 | 1 | 2 | 2 |
| | 3 | 3 | 1 | 1 | 2 | 2 |
| | 4 | 3 | 1 | 2 | 2 | 4 |