Chromaticity Coordinates

- Tristimulus values X, Y, Z specify a color's:
  - Lightness - light or dark
  - Huge - red, orange, yellow, green, blue, purple
  - Saturation - pink-red; pastel-fluorescent; baby blue-deep blue
- The *chromaticity* specifies the hue and saturation, but not the lightness.

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

Properties of Chromaticity Coordinates

$$
\begin{aligned}
x &= \frac{X}{X+Y+Z} \\
y &= \frac{Y}{X+Y+Z} \\
z &= \frac{Z}{X+Y+Z}
\end{aligned}
$$

- $x + y + z = 1$ - Third component can always be computed from first two.
- Typically, (x, y) are specified
- Let α be any positive constant. Then (X, Y, Z) and $(\alpha X, \alpha Y, \alpha Z)$ have the same chromaticity coordinates.
- Projection property: Straight lines in XYZ map to straight lines in (x, y).

Projection Property of Chromaticity Coordinates

- Fact: Straight lines in (X, Y, Z) space project to straight lines in (x, y) chromaticity space.

  Proof:

  - Let $C_1 = (X_1, Y_1, Z_1)$ and $C_2 = (X_2, Y_2, Z_2)$ be two different colors, and let $C_3 = (X_3, Y_3, Z_3)$ fall on a line connecting C_1 and C_2.
  - In this case, we know that

$$\begin{aligned} C_3 &= \alpha C_1 + \beta C_2 \\ (X_3, Y_3, Z_3) &= \alpha(X_1, Y_1, Z_1) + \beta(X_2, Y_2, Z_2) \end{aligned}$$

    where

$$\alpha + \beta = 1$$

  - In order to show that (x_3, y_3) falls on a straight line connecting (x_1, y_1) and (x_2, y_2), we must show that

$$(x_3, y_3) = \alpha'(x_1, y_1) + \beta'(x_2, y_2)$$

    where

$$\alpha' + \beta' = 1$$

Projection Property (2)

$$
\begin{aligned}
& (x_3, y_3) \\
= \; & \left(\frac{\alpha X_1 + \beta X_2}{X_3 + Y_3 + Z_3}, \frac{\alpha Y_1 + \beta Y_2}{X_3 + Y_3 + Z_3} \right) \\
= \; & \left(\frac{\alpha X_1}{X_3 + Y_3 + Z_3}, \frac{\alpha Y_1}{X_3 + Y_3 + Z_3} \right) \\
& + \left(\frac{\beta X_2}{X_3 + Y_3 + Z_3}, \frac{\beta Y_2}{X_3 + Y_3 + Z_3} \right) \\
= \; & \frac{X_1 + Y_1 + Z_1}{X_3 + Y_3 + Z_3} \left(\frac{\alpha X_1}{X_1 + Y_1 + Z_1}, \frac{\alpha Y_1}{X_1 + Y_1 + Z_1} \right) \\
& + \frac{X_2 + Y_2 + Z_2}{X_3 + Y_3 + Z_3} \left(\frac{\beta X_2}{X_2 + Y_2 + Z_2}, \frac{\beta Y_2}{X_2 + Y_2 + Z_2} \right) \\
= \; & \alpha \frac{X_1 + Y_1 + Z_1}{X_3 + Y_3 + Z_3} (x_1, y_1) + \beta \frac{X_2 + Y_2 + Z_2}{X_3 + Y_3 + Z_3} (x_2, y_2) \\
= \; & \alpha' (x_1, y_1) + \beta' (x_2, y_2)
\end{aligned}
$$

Projection Property (3)

- Then α' and β' are given by

$$\alpha' = \frac{\alpha(X_1+Y_1+Z_1)}{\alpha(X_1+Y_1+Z_1)+\beta(X_2+Y_2+Z_2)}$$

$$\beta' = \frac{\beta(X_2+Y_2+Z_2)}{\alpha(X_1+Y_1+Z_1)+\beta(X_2+Y_2+Z_2)}$$

So we have that

$$\alpha' + \beta' = 1$$

QED

Chromaticity Diagrams

- Compute the chromaticity of a pure spectral line at wavelength λ_0.
- The XYZ values are given by

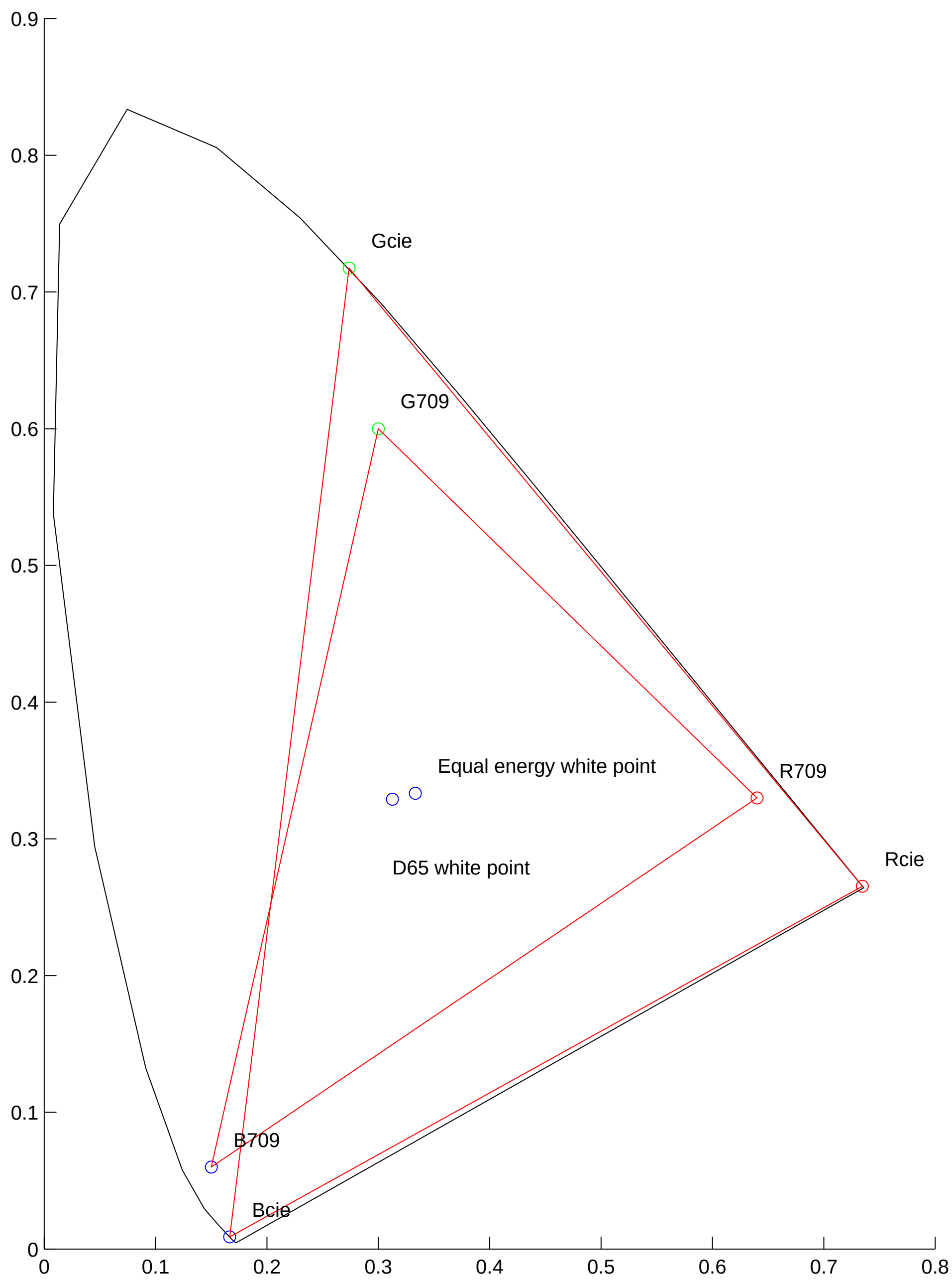
$$
\begin{aligned}
X &= \int_0^\infty \delta(\lambda - \lambda_0) x_0(\lambda) d\lambda = x_0(\lambda_0) \\
Y &= \int_0^\infty \delta(\lambda - \lambda_0) y_0(\lambda) d\lambda = y_0(\lambda_0) \\
Z &= \int_0^\infty \delta(\lambda - \lambda_0) z_0(\lambda) d\lambda = z_0(\lambda_0)
\end{aligned}
$$

- So the chromaticity of a spectral line at wavelength λ is given by

$$
(x, y) = \left(\frac{x_0(\lambda)}{x_0(\lambda) + y_0(\lambda) + z_0(\lambda)}, \frac{y_0(\lambda)}{x_0(\lambda) + y_0(\lambda) + z_0(\lambda)} \right)
$$

- Plot this parametric curve in (x, y) as a function of λ.

Chromaticity Diagram



- Horse shoe shape results form XYZ color matching functions

Chromaticity Diagrams

- Linear combinations of colors form straight lines.
- Any color in the interior (i.e. convex hull) of the "horse shoe" can be achieved through the linear combination of two pure spectral colors.
- The straight line connecting red and blue is referred to as "line of purples".
- RGB primaries form a triangular color gamut.
- The color white falls in the center of the diagram.

What is White Point?

- What is white point?
- There are three major functions for the concept of white point.
  - *Calibration:* Absolute scaling of (r, g, b) values required for calibrated image data. This determines the color associated with $(r, g, b) = (1, 1, 1)$.
  - *Color constancy:* Color of illuminant in scene. By changing white point, one can partially compensate for changes due to illumination color. (camcorders)
  - *Gammut mapping:* Color of paper in printing applications. Color of paper is brightest white usually possible. Should a color photocopier change the color of the paper? Usually no.
- We will focus on use of white point for calibration.

Defining White Point?

- Ideally white point specifies the spectrum of the color white.

$$I_w(\lambda)$$

- This specifies XYZ coordinates

$$\begin{aligned} X_w &= \int_0^\infty x_0(\lambda)\, I_w(\lambda) d\lambda \\ Y_w &= \int_0^\infty y_0(\lambda)\, I_w(\lambda) d\lambda \\ Z_w &= \int_0^\infty z_0(\lambda)\, I_w(\lambda) d\lambda \end{aligned}$$

which in turn specifies chromaticity components

$$\begin{aligned} x_w &= \frac{X_w}{X_w + Y_w + Z_w} \\ y_w &= \frac{Y_w}{X_w + Y_w + Z_w} \end{aligned}$$

- Comments
  - White point is usually specified in chromaticity.
  - Knowing (x_w, y_w) does not determine $I_w(\lambda)$.

Typical White Points

- Equal energy white:

$$I_{EE}(\lambda) = 1$$

$$(x_{EE}, y_{EE}, z_{EE}) = (1/3, 1/3, 1/3)$$

- D65 illuminant (specified for PAL):

$$I_{65}(\lambda) = \text{Natural Sun Light}$$

$$(x_{65}, y_{65}, z_{65}) = (0.3127, 0.3290, 0.3583)$$

- C illuminant (specified for NTSC):

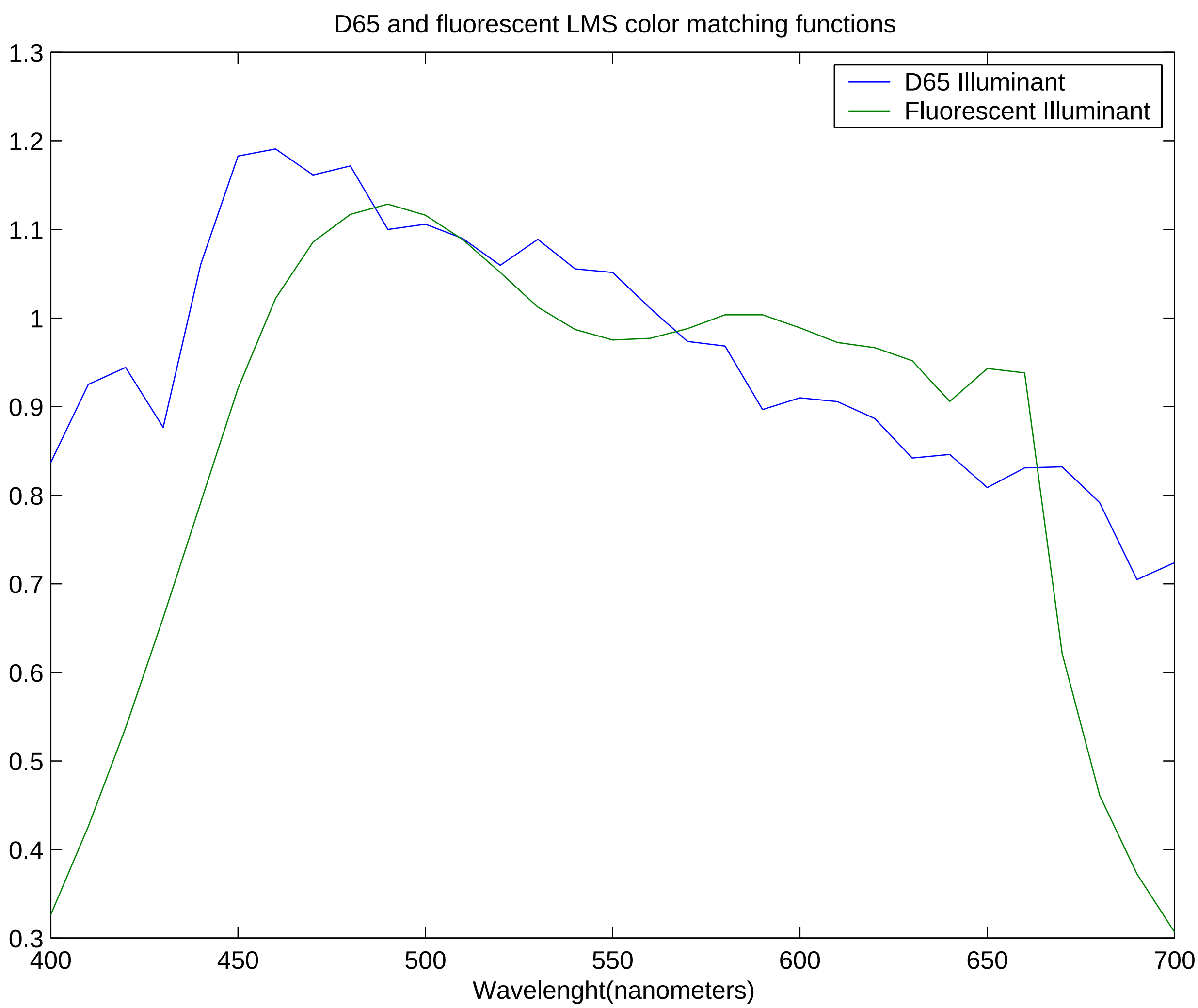
$$I_c(\lambda) = \text{not defined}$$

$$(x_c, y_c, z_c) = (0.310, 0.316, 0.374)$$

- Comments:
  - Equal energy white is not commonly used.
  - C was the original standard for NTSC video.
  - $D65$ has become the dominant standard.
  - $D65$ corresponds to a color temperature of 6500^oK.

Two Example Illuminants

- Examples of D65 and Fluorescent Illuminants.



White Point Correction

- Standard color matching functions assume equal energy white point
  - Any standard color matching function assumes unit area normalization.

$$\begin{aligned} 1 &= \int_0^\infty r_0(\lambda) d\lambda \\ 1 &= \int_0^\infty g_0(\lambda) d\lambda \\ 1 &= \int_0^\infty b_0(\lambda) d\lambda \end{aligned}$$

  - Therefore:

$$I_{EE}(\lambda) = 1 \Rightarrow (r, g, b) = (1, 1, 1)$$

- White point corrected/gamma corrected data is compute as:

$$\begin{aligned} \tilde{r} &\triangleq \left(\frac{r}{r_{wp}}\right)^{1/\gamma} \\ \tilde{g} &\triangleq \left(\frac{g}{g_{wp}}\right)^{1/\gamma} \\ \tilde{b} &\triangleq \left(\frac{b}{b_{wp}}\right)^{1/\gamma} \end{aligned}$$

  - So,

$$(\tilde{r}, \tilde{g}, \tilde{b}) = (1, 1, 1) \Rightarrow (r, g, b) = (r_{wp}, g_{wp}, b_{wp})$$

  where (r_{wp}, g_{wp}, b_{wp}) is the desired white point.

Typical RGB Color Primaries

- NTSC 601 standard primaries:

$$\begin{aligned}(x_r, y_r) &= (0.67, 0.33)\\(x_g, y_g) &= (0.21, 0.71)\\(x_b, y_b) &= (0.14, 0.08)\end{aligned}$$

  – These color primaries are not typically used anymore.

- PAL standard primaries:

$$\begin{aligned}(x_r, y_r) &= (0.64, 0.33)\\(x_g, y_g) &= (0.29, 0.60)\\(x_b, y_b) &= (0.15, 0.06)\end{aligned}$$

  – PAL is the TV standard used in Europe

- ITU-R BT.709 standard primaries:

$$\begin{aligned}(x_r, y_r, z_r) &= (0.6400, 0.3300, 0.0300)\\(x_g, y_g, z_g) &= (0.3000, 0.6000, 0.1000)\\(x_b, y_b, z_b) &= (0.1500, 0.0600, 0.7900)\end{aligned}$$

  – More saturated then 601 primaries.

  – Most commonly used primary colors for display monitors and TV's.

Example: 601 Color Primaries With EE White Point

- We need to find a transformation $\mathbf{M}$ so that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M} \begin{bmatrix} r_{lin} \\ g_{lin} \\ b_{lin} \end{bmatrix}$$

  - $(r_{lin}, g_{lin}, b_{lin})$ are linear (i.e. $\gamma = 1$).
  - Columns of $\mathbf{M}$ are proportional to color primaries.
  - Rows of $\mathbf{M}$ sum to 1 $\Rightarrow$ equal energy white point.

- Therefore, M must have the following form for some α_1, α_2, and α_3.

$$\mathbf{M} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

Example Continued: 601 Color Primaries With EE White Point

- In order to have an EE white point, the values of α_1, α_2, and α_3 must satisfy the equation.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \mathbf{M} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

- This results in $[\alpha_1, \alpha_2, \alpha_3] = (0.9867, 0.8148, 1.1985)$.
- Substituting in α_1, α_2, and α_3 yields:

$$\mathbf{M} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} 0.9867 & 0 & 0 \\ 0 & 0.8148 & 0 \\ 0 & 0 & 1.1985 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6611 & 0.1711 & 0.1678 \\ 0.3256 & 0.5785 & 0.0959 \\ 0 & 0.0652 & 0.9348 \end{bmatrix}$$

Example: 601 Color Primaries With C White Point

- Find a transformation $\mathbf{M}$ so that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M} \begin{bmatrix} r_{lin} \\ g_{lin} \\ b_{lin} \end{bmatrix}$$

  where

  – Columns of $\mathbf{M}$ are proportional to color primaries.

  – Rows of $\mathbf{M}$ sum to $[0.310, 0.316, 0.374] \times constant$.

  – Middle rows of $\mathbf{M}$ sum to 1 $\Rightarrow$ unit luminance.

- Solve the equation

$$\frac{1}{0.316} \begin{bmatrix} 0.310 \\ 0.316 \\ 0.374 \end{bmatrix} = \begin{bmatrix} 0.9810 \\ 1 \\ 1.1835 \end{bmatrix} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

- This results in $[\alpha_1, \alpha_2, \alpha_3] = (0.9060, 0.8259, 1.4327)$, and

$$\mathbf{M} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} 0.9060 & 0 & 0 \\ 0 & 0.8259 & 0 \\ 0 & 0 & 1.4327 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6070 & 0.1734 & 0.2006 \\ 0.2990 & 0.5864 & 0.1146 \\ 0 & 0.0661 & 1.1175 \end{bmatrix}$$

The International Color Consortium (ICC) (www.color.org)

- Sets industry standards for color management
- ICC color management standard
  - Uses point to point transformation techniques to calibrate color capture and rendering devices with the best possible fidelity.
  - Based on Apples ColorSync system.
  - Requires color profiles for each input and output device.
  - Requires that each image have an associated color profile.
  - But most image file formats do not support color profile embedding.
  - Difficult for non-professionals to use.
- ICC color management system does not specify a single universal color space for interchange of data.

sRGB: The New Industry Color Standard (www.color.org/sRGB.html)

- Industry standard color space proposed by Hewlett-Packard and Microsoft through the ICC organization.
- Defines a standard color space for images in RGB format.
- Basic sRGB standard:
  - Gamma corrected format with $\gamma = 2.2.$ (approxiately)
  - 709 Primaries
  - D65 white point

Converting From sRGB to XYZ

- First convert from gamma corrected to linear sRGB. (approximate)

$$\begin{aligned} r_{lin} &= \left(\frac{r}{255}\right)^{2.2} \\ g_{lin} &= \left(\frac{g}{255}\right)^{2.2} \\ b_{lin} &= \left(\frac{b}{255}\right)^{2.2} \end{aligned}$$

- Make sure that $(r_{lin}, g_{lin}, b_{lin})$ are stored using floating point precision.
- Then convert from linear sRGB to XYZ using linear transformation.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M} \begin{bmatrix} r_{lin} \\ g_{lin} \\ b_{lin} \end{bmatrix}$$

- How do we compute $\mathbf{M}$?

sRGB Linear Color Transformation

- Find a transformation $\mathbf{M}$ so that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M} \begin{bmatrix} r_{lin} \\ g_{lin} \\ b_{lin} \end{bmatrix}$$

  - Columns of $\mathbf{M}$ are proportional to color primaries.
  - Rows of $\mathbf{M}$ sum to $[0.3127, 0.3290, 0.3583] \times constant$.
  - Middle row of $\mathbf{M}$ sums to 1 $\Rightarrow$ unit luminance.

- Solve the equation

$$\begin{aligned} \frac{1}{0.3290} \begin{bmatrix} 0.3127 \\ 0.3290 \\ 0.3583 \end{bmatrix} &= \begin{bmatrix} 0.9505 \\ 1 \\ 1.0891 \end{bmatrix} \\ &= \begin{bmatrix} 0.6400 & 0.3000 & 0.1500 \\ 0.3300 & 0.6000 & 0.0600 \\ 0.0300 & 0.1000 & 0.7900 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \end{aligned}$$

- This results in $[\alpha_1, \alpha_2, \alpha_3] = (0.6444, 1.1919, 1.2032)$, and

$$\mathbf{M} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix}$$

Summary of sRGB to XYZ Transform

- First convert to linear sRGB. (approximate)

$$
\begin{aligned}
r_{lin} &= \left(\frac{r}{255}\right)^{2.2} \\
g_{lin} &= \left(\frac{g}{255}\right)^{2.2} \\
b_{lin} &= \left(\frac{b}{255}\right)^{2.2}
\end{aligned}
$$

- Then convert from linear sRGB to XYZ using floating point operations

$$
\begin{aligned}
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} &= \mathbf{M} \begin{bmatrix} r_{lin} \\ g_{lin} \\ b_{lin} \end{bmatrix} \\
&= \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} r_{lin} \\ g_{lin} \\ b_{lin} \end{bmatrix}
\end{aligned}
$$

Converting From XYZ to sRGB

- First convert from XYZ to linear sRGB using floating point operations

$$\begin{aligned}\begin{bmatrix} r_{lin} \\ g_{lin} \\ b_{lin} \end{bmatrix} &= \mathbf{M}^{-1}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\ &= \begin{bmatrix} 3.2410 & -1.5374 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0570 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}\end{aligned}$$

- Then gamma correct using $\gamma = 2.2$. (approximate)

$$\begin{aligned} r &= 255 * (r_{lin})^{\frac{1}{2.2}} \\ g &= 255 * (g_{lin})^{\frac{1}{2.2}} \\ b &= 255 * (b_{lin})^{\frac{1}{2.2}} \end{aligned}$$

Analog NTSC Standard

- Assume:
  - 601 color primaries
  - C white point
- Then the "luminance" component Y is given by

$$Y = 0.2990\, r_{lin} + 0.5864\, g_{lin} + 0.1146\, b_{lin}$$

- By convention, NTSC transforms are performed on the gamma corrected (r, g, b).
- So, $\tilde{Y}$ is given by

$$\tilde{Y} = 0.2990\, r + 0.5864\, g + 0.1146\, b$$

Analog NTSC Color Spaces

- Then, define the YPrPb coordinates system as

$$\begin{bmatrix} \tilde{Y} \\ Pb \\ Pr \end{bmatrix} = \begin{bmatrix} \tilde{Y} \\ \tilde{b} - \tilde{Y} \\ \tilde{r} - \tilde{Y} \end{bmatrix}$$

- Then, YUV coordinates are defined as

$$\begin{bmatrix} \tilde{Y} \\ U \\ V \end{bmatrix} = \begin{bmatrix} \tilde{Y} \\ Pb/2.03 \\ Pr/1.14 \end{bmatrix}$$

- Then, YIQ is a 33^o rotation of the UV color space

$$\begin{bmatrix} \tilde{Y} \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\sin 33^o & \cos 33^o \\ 0 & \cos 33^o & \sin 33^o \end{bmatrix} \begin{bmatrix} Y \\ U \\ V \end{bmatrix}$$

$$= \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{bmatrix} \begin{bmatrix} \tilde{r} \\ \tilde{g} \\ \tilde{b} \end{bmatrix}$$

Comments on Analog NTSC Color Standard

- Technically, YPbPr, YUV and YIQ assume NTSC 601 primaries with C white point.
- Same transformations may be used with other white point and color primaries.
- In practice, people usually use the Rec. 701 primaries with D65 white point.
- All transformations are performed on gamma corrected RGB.
- Nominal bandwidth for Y, I, and Q channels are 4.2MHz, 1.5MHz, and 0.6MHz.
- This chromaticity coordinate system is approximately an opponent color system.

Digital NTSC Color Standard

- Assuming that (r, g, b) are
  - Gamma corrected with $\gamma = 2.2$.
  - Scaled to the range 0 to 255
- First compute the "luminance" component.

$$\tilde{Y} = 0.326\, r + 0.578\, g + 0.096\, b$$

- The values of YCrCb are then given by

$$\begin{bmatrix} Y_d \\ c_b \\ c_r \end{bmatrix} = \begin{bmatrix} \frac{219\tilde{Y}}{255} + 16 \\ \frac{112(b-\tilde{Y})}{0.886*255} + 128 \\ \frac{112(r-\tilde{Y})}{0.701*255} + 128 \end{bmatrix}$$

- Complete transformation assuming gamma corrected (r, g, b) in the range of 0 to 255.

$$\begin{bmatrix} Y_d \\ c_b \\ c_r \end{bmatrix} = \begin{bmatrix} 16 \\ 128 \\ 128 \end{bmatrix} + \begin{bmatrix} 0.2568 & 0.5036 & 0.0984 \\ -0.1482 & -0.2907 & 0.4389 \\ 0.4392 & -0.3674 & -0.0718 \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

- Again, transformations may be used with other color primaries and white points.