EE637 Spring 2001
Exam #1 Solution

1)

a) Set $x(m,n) = \delta(m,n)$

$$h(m,n) = \delta(m,n-1) + 2\delta(m-1,n) - 2\delta(m+1,n) - \delta(m,n+1) + 3\delta(m,n)$$

$$\begin{array}{ccc}
\xrightarrow{m} & & \\
n\downarrow & & \\
\end{array}
\qquad
\begin{array}{ccc}
 & -1 & \\
-2 & \boxed{3} & 2 \\
 & 1 &
\end{array}$$

b) $$H(e^{j0}, e^{j0}) = \sum_{m,n} h(m,n) = 3$$

c)

$$\begin{array}{ccccc}
0 & 0 & 0 & -3 & +2 \\
0 & 0 & -3 & 0 & 5 \\
0 & -3 & 0 & 3 & 5 \\
-3 & 0 & 3 & 3 & 5 \\
1 & 4 & 4 & 4 & 6
\end{array}$$

2)

a) $$Y(z_1, z_2) = X(z_1, z_2) + \frac{1}{2} z_1^{-1} z_2^{-1} Y(z_1, z_2)$$

$$H(z_1, z_2) = \frac{1}{(1 - \frac{1}{2} z_1^{-1} z_2^{-1})}$$

b)

$$\begin{array}{ccccc}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 16 & 0 & 0 \\
0 & 0 & 0 & 8 & 0 \\
0 & 0 & 0 & 0 & 4
\end{array}$$

3) a)

$$\frac{1}{1-az^{-1}} \overset{z\text{-trans}}{\Longleftrightarrow} u(n)\,a^n$$

$$\frac{1-az}{1-az^{-1}} \overset{z\text{-tran}}{\Longleftrightarrow} \left.\begin{array}{l} u(n)\,a^n \\ -a\,u(n+1)\,a^{n+1} \end{array}\right\} = h(n)$$

$$u(n+1)\,a^{n+1} = \delta(n+1) + u(n)\,a^{n+1}$$

So

$$h(n) = u(n)\,a^n - a\,\delta(n+1) - a^2 u(n)\,a^n$$

$$= (1-a^2)\,u(n)\,a^n - a\,\delta(n+1)$$

b)

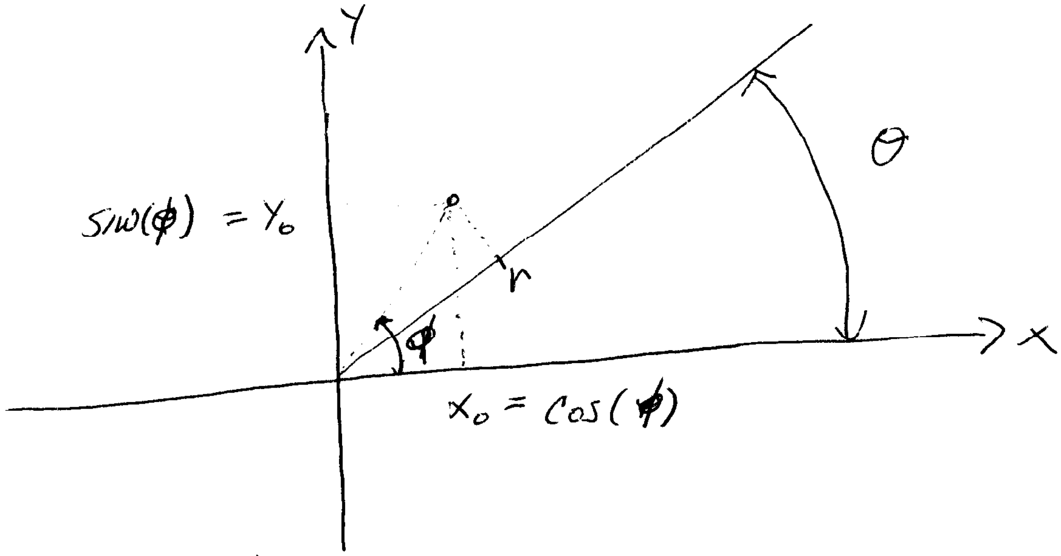
$$H(e^{j\omega}) = \frac{(1-ae^{j\omega})}{(1-ae^{-j\omega})}$$

$$|H(e^{j\omega})|^2 = \frac{|1-ae^{j\omega}|^2}{|1-ae^{-j\omega}|^2} = 1$$

$$S_y(e^{j\omega}) = 1 \cdot S_x(e^{j\omega}) = 1$$

4)

a)



$$r = \cos(\phi - \theta) = \cos(\theta - \phi)$$

So

$$p_\theta(t) = \delta(t - \cos(\phi - \theta))$$
$$= \delta(t - \cos(\theta - \phi))$$

b) $$P_\theta(\rho) = e^{-j\rho\cos(\theta-\phi)}$$

c) $F(u, v) =$

$$\iint_{\mathbb{R}\,\mathbb{R}} \delta(x - \cos\phi, y - \sin\phi)\, e^{j(ux + vy)}\, dx\,dy$$

$$= e^{-j(u\cos\phi + v\sin\phi)}$$

d) Fourier Slice Theorem Says

$$F(\rho\cos\theta, \rho\sin\theta) = P_\theta(\rho)$$

We know that

$$P_\theta(\rho) = e^{-j\rho\cos(\theta-\phi)}$$

$$F(\rho\cos\theta, \rho\sin\theta) =$$

$$e^{-j(\rho\cos\theta\cos\phi + \rho\sin\theta\sin\phi)}$$

We know that

$$\cos\theta\cos\phi + \sin\theta\sin\phi = \cos(\theta-\phi)$$

$$F(\rho\cos\theta, \rho\sin\phi)$$

$$= e^{-j\rho\cos(\theta-\phi)}$$

$$= P_\theta(\rho)$$