EE 637 Final Exam

May 2, Spring 2001

Name: ______________________________________

Instructions:

- Follow all instructions carefully!
- This is a 120 minute exam containing **five** problems.
- You may **only** use your brain and a pencil (or pen) to complete this exam. You **may not** use your book, notes or a calculator.

Good Luck.

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Problem 1.(30pt)

Let X_n be a discrete random variable which takes values on the set $\{0, 1, \cdots, 5\}$, and let

$$P\{X_n = k\} = p_k$$

where

$$(p_0, p_1, p_2, p_3, p_4, p_5) = (0.1, 0.04, 0.2, 0.06, 0.35, 0.25)$$

a) Draw and fully label the binary tree used to form a Huffman code for X_n.

b) Write out the Huffman codes for the six symbols $0, 1, \cdots, 5$

c) Compute the expected code length for your Huffman code.

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Problem 2.(30pt)

Let X be a scalar random variable and Z be a $M \times 1$ random vector. Assume that X and Z are zero mean with

$$\begin{aligned} b &= E[XZ] \\ R &= E[ZZ^t] \ . \end{aligned}$$

Further define the estimator

$$\hat{X} = \theta Z$$

where θ is a $1 \times M$ parameter vector.

a) **Derive** an expression for the value of θ that results in the minimum mean squared error **linear** estimator of X given Z.

b) Is the minimum mean squared error estimator of X always a linear function of Z? Justify your answer.

c) Further assume that (X, Z) are jointly Gaussian. What estimator, $\hat{X} = T(Z)$, minimizes the following expression?

$$E[|X - \hat{X}|]$$

Justify your answer.

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Problem 3.(50pt)

Consider the 1-D multilevel error diffusion algorithm specified by the equations

$$\begin{aligned} z_n &= Q(y_n) \\ e_n &= y_n - z_n \\ y_n &= x_n + e_{n-1} \end{aligned}$$

where x_n is the input, z_n is the output, and $Q(\cdot)$ is a uniform quantizer with step size Δ.

a) Draw a block diagram for the error diffusion filter described by these equations. Make sure you clearly label all the signals.

A conventional approach is to model the quantizer as adding i.i.d. noise with mean zero and variance $\frac{1}{12\Delta^2}$. So in this case, the model would be

$$Q(y_n) = y_n + w_n$$

where w_n are i.i.d. with $E[w_n] = 0$ and $E[(w_n)^2] = \frac{1}{12\Delta^2}$.

b) Use this model to express the output, z_n, in terms of the input, x_n, and the white noise, w_n.

c) Compute the display error $d_n = z_n - x_n$ in terms of the input, x_n, and the white noise, w_n.

d) Compute the power spectrum of the display error d_n.

e) Explain what display error power spectrum is most desirable for the halftoning of images, and explain why it is desirable.

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Problem 4.(30pt)

Let $x(m,n)$ and $y(m,n)$ be the input to a monitor with an unknown value of γ. In particular,

$$\begin{aligned} x(m,n) &= \begin{cases} 255 & \text{if } m \text{ is even} \\ 0 & \text{if } m \text{ is odd} \end{cases} \\ y(m,n) &= C \end{aligned}$$

The display has a resolution of 100 dots per inch, and the two images are viewed at a distance d. The value of C is adjusted so that the two images look similar when d is large.

a) Sketch the 2-D frequency spectrum of the observed image corresponding to $x(m,n)$ and label the axis of your plot in units of cycles per degree.

b) Explain why the two images would look similar at a large distance. Approximately how large should d be for the two images to appear similar?

c) Compute an expression for the value of γ in terms of the value C.

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Problem 5.(40pt)

Consider a 1-D MRI system with static field M_0 and magnetic field gradient $G(t)$. The frequency of the radiated signal at position x is then

$$f(t) = xLG(t) + LM_0$$

where L is the Lamar constant. Denote the amplitude of the response at position x as $A(x)$.

a) Compute the phase, $\phi(t)$, of the radiated signal.

b) Compute the expression, $r(x,t)$, for the signal radiated from a small interval $[x, x+dx]$.

c) Compute the expression for the total received signal, $s(t)$.

d) Explain how the amplitudes $A(x)$ can be compute from the observed signals $s(t)$.

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