EE 637 Midterm Exam #1

Session 17

February 16, Spring 2001

Name: ____________________________________

Instructions:

- Follow all instructions carefully!
- This is a 50 minute exam containing **four** problems.
- You may **only** use your brain and a pencil (or pen) to complete this exam. You **may not** use your book, notes or a calculator.
- This is also homework #1. Please keep it. Complete it as a homework, and hand it (or mail it) in by Session 20 (Friday February 23).

Good Luck.

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Problem 1.(30pt)

Consider the following discrete space system with input $x(m,n)$ and output $y(m,n)$.

$$y(m,n) = x(m,n-1) + 2x(m-1,n) - 2x(m+1,n) - x(m,n+1) + 3x(m,n)$$

a) Calculate the point spread function for this system. Write out the values of the point spread function on your paper. Clearly indicate the direction of m and n, and draw a box around the $(0,0)$ position.

b) Calculate the value of $H(e^{j0}, e^{j0})$ where $H(e^{j\mu}, e^{j\nu})$ is the transfer function of the system.

c) Calculate the output $y(m,n)$ for the following input using a free boundary condition.

$$\begin{array}{c} \\ n \\ \downarrow \end{array} \quad \begin{array}{ccccc} & & m \rightarrow & & \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{array}$$

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Problem 2.(20pt)

Consider the following discrete space system with input $x(m,n)$ and output $y(m,n)$.

$$y(m,n) = x(m,n) + (1/2)y(m-1,n-1)$$

a) Compute the transfer function of the system

$$H(z_1,z_2) = \frac{Y(z_1,z_2)}{X(z_1,z_2)}$$

b) Compute the output $y(m,n)$ for the following input using a free boundary condition.

$$\begin{array}{cc} & m \rightarrow \\ n & \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \\ \downarrow & \end{array}$$

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Problem 3.(20pt)

Let X_n be a discrete time random process composed of i.i.d. Gaussian random variables with distribution $N(0,1)$. Let $H(z)$ be a stable discrete time filter with transfer function

$$H(z) = \frac{1 - az}{1 - az^{-1}}$$

where $|a| < 1$.

a) Compute the impulse response h_n for the system.

b) Compute $S_Y(e^{j\omega})$ the power spectrum of the random process

$$Y_n = h_n * X_n$$

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Problem 4.(30pt)

Consider the continuous-space signal

$$f(x, y) = \delta\left(x - \cos\phi, y - \sin\phi\right)$$

where $-\pi < \phi < \pi$ is an angle.

a) Calculate the function $p_\theta(t)$ which is known as the forward projection or equivalently the Radon transform of the signal $f(x, y)$.

b) Calculate the CTFT of the forward projections.

$$P_\theta(\rho) = CTFT\{p_\theta(t)\}$$

c) Calculate the CSFT of $f(x, y)$.

d) Show that the Fourier Slice Theorem holds for this signal.

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