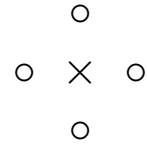


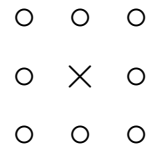
2-D Neighborhoods

- 4-point neighborhood



$$\partial(i, j) = \{(i + 1, j), (i - 1, j), (i, j + 1), (i, j - 1)\}$$

- 8-point neighborhood



$$\partial(i, j) = \{(i+1, j), (i-1, j), (i, j+1), (i, j-1), (i+1, j+1), (i-1, j+1), (i+1, j-1), (i-1, j-1)\}$$

- More generally, a *Neighborhood System* is any mapping with the two properties that:

1. For all $s \in S$, $s \notin \partial s$
2. For all $r \in S$, $r \in \partial s \Rightarrow s \in \partial r$

Boundary Conditions

- How do you process pixels on the boundary of an image??
- Consider the following example using a 4-point neighborhood

A small example image

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>

4 neighbors of l

	l_1	
l_4	l	l_2
	l_3	

- Free boundary condition

$$\partial l = \{h, p, k\}$$

$$\partial p = \{l, o\}$$

- Toriodal boundary condition (asteroids)

$$\partial l = \{h, i, p, k\}$$

$$\partial p = \{l, m, d, o\}$$

- Reflective boundary condition

$$l_1 = h, \quad l_2 = k, \quad l_3 = p, \quad l_4 = k$$

$$p_1 = l, \quad p_2 = o, \quad p_3 = l, \quad p_4 = o$$

Connected Sets

- Let $c(s)$ be a neighborhood system such that $c(s) \subset \partial s$.
 - For example, $c(s) = \{r \in \partial s : X_r = X_s\}$
 - Computation of $c(s)$ might be very difficult, but we won't worry about that now.
- Definition: A region $R \subset S$ is said to be connected under neighborhood system $c(s)$ if for all $s, r \in R$ there exists a sequence of M pixels, $s_1, \dots, s_M$ such that

$$s_1 \in c(s), s_2 \in c(s_1), \dots, s_M \in c(s_{M-1}), r \in c(s_M)$$

i.e. there is a connected path from s to r .

Example of Connect Sets

- Consider the following image X_s

1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
0	0	0	1	1	1
0	0	0	1	1	1
0	0	0	1	1	1

$$S_1 = \{s : X_s = 1\}$$

$$S_0 = \{s : X_s = 0\}$$

- Define $c(s) = \{r \in \partial s : X_r = X_s\}$
- Result
 - 4-point neighborhood $\Rightarrow S_0$ and S_1 are not connected sets
 - 8-point neighborhood $\Rightarrow S_0$ and S_1 **are** connected!

Region Growing

- Idea - grow out region from a seed point s_0
- Assume that $c(s)$ is given

```
ClassLabel = 1
Initialize  $Y_r = 0$  for  $r \in S$ 
ConnectedSet( $s_0, Y, \textit{ClassLabel}$ ) {
     $Y_{s_0} = \textit{ClassLabel}$ 
     $B = \{s\}$ 
    While  $B$  is not empty {
         $s \leftarrow$  any element of  $B$ 
         $B \leftarrow B - \{s\}$ 
         $Y_s \leftarrow \textit{ClassLabel}$ 
         $B \leftarrow B \cup \{r : r \in c(s) \text{ and } Y_r = 0\}$ 
    }
    return( $Y$ )
}
```

Region Growing Example (1)

The list of
 $(i, j) \in B$
(0,0)

The image X

	j					
	0	1	2	3	4	
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j					
		0	1	2	3	4	
i	0	1	0	0	0	0	
	1	0	0	0	0	0	
	2	0	0	0	0	0	
	3	0	0	0	0	0	
	4	0	0	0	0	0	

Region Growing Example (2)

The list of
 $(i, j) \in B$
(1,0)

The image X

	j				
	0	1	2	3	4
i	0	1	0	0	0
	1	1	0	0	0
	2	0	1	0	0
	3	0	1	0	0
	4	0	1	0	1

The segmentation Y

	j				
	0	1	2	3	4
i	0	1	0	0	0
	1	1	0	0	0
	2	0	0	0	0
	3	0	0	0	0
	4	0	0	0	0

Region Growing Example (3)

The list of
 $(i, j) \in B$
 (1,1)

The image X

	j				
	0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

	j				
	0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0

Region Growing Example (4)

The list of
 $(i, j) \in B$
(2,1)

The image X

	j					
	0	1	2	3	4	
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

	j					
	0	1	2	3	4	
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	0	0	0
	3	0	0	0	0	0
	4	0	0	0	0	0

Region Growing Example (5)

The list of
 $(i, j) \in B$
(3,1)
(2,2)

The image X

	j					
	0	1	2	3	4	
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	0	0	0
	4	0	1	0	0	1

The segmentation Y

	j					
	0	1	2	3	4	
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	0	0	0
	4	0	0	0	0	0

Region Growing Example (6)

The list of
 $(i, j) \in B$
 (4,1)
 (3,2)
 (2,2)

		The image X				
		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

		The segmentation Y				
		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Region Growing Example (7)

The list of
 $(i, j) \in B$
(3,2)
(2,2)

The image X

	j					
	0	1	2	3	4	
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

	j					
	0	1	2	3	4	
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Region Growing Example (8)

The list of
 $(i, j) \in B$
 $(2,2)$

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Region Growing Example (9)

The list of
 $(i, j) \in B$
 empty

The image X

	j				
	0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

	j				
	0	1	2	3	4
i 0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	0

Connected Components Extraction

- Iterate through each pixel in the image.
- Extract connected set for each unlabeled pixel.

$ClassLabel = 1$

Initialize $Y_r = 0$ for $r \in S$

For each $s \in S$ {

if($Y_s \neq 0$) {

ConnectedSet($s, Y, ClassLabel$)

$ClassLabel \leftarrow ClassLabel + 1$

}

}

Connected Components Extraction Example (1)

$s = (i, j); \textit{ClassLabel}$
 $(0, 0); 1$

The image X

		j					
		0	1	2	3	4	
i	0	1	0	0	0	0	
	1	1	1	0	0	0	
	2	0	1	1	0	0	
	3	0	1	1	0	0	
	4	0	1	0	0	1	

The segmentation Y

		j					
		0	1	2	3	4	
i	0	1	0	0	0	0	
	1	1	1	0	0	0	
	2	0	1	1	0	0	
	3	0	1	1	0	0	
	4	0	1	0	0	0	

Connected Components Extraction Example (2)

$s = (i, j); ClassLabel$
 $(0, 1); 2$

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	2	2	2	2
	1	1	1	2	2	2
	2	0	1	1	2	2
	3	0	1	1	2	2
	4	0	1	2	2	0

Connected Components Extraction Example (4)

$s = (i, j); \textit{ClassLabel}$
 $(4, 4); 4$

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	2	2	2	2
	1	1	1	2	2	2
	2	3	1	1	2	2
	3	3	1	1	2	2
	4	3	1	2	2	4