2-D Neighborhoods

- 4-point neighborhood

$$\begin{array}{ccc} & \circ & \\ \circ & \times & \circ \\ & \circ & \end{array}$$

$$\partial(i, j) = \{(i+1, j), (i-1, j), (i, j+1), (i, j-1)\}$$

- 8-point neighborhood

$$\begin{array}{ccc} \circ & \circ & \circ \\ \circ & \times & \circ \\ \circ & \circ & \circ \end{array}$$

$$\partial(i, j) = \{(i+1, j), (i-1, j), (i, j+1), (i, j-1)(i+1, j+1), (i-1, j+1), (i+1, j-1), (i-1, j-1)\}$$

- More generally, a *Neighborhood System* is any mapping with the two properties that:

  1. For all $s \in S$, $s \notin \partial s$
  2. For all $r \in S$, $r \in \partial s \Rightarrow s \in \partial r$

Boundary Conditions

- How do you process pixels on the boundary of an image??
- Consider the following example using a 4-point neighborhood

A small example image

$$\begin{array}{cccc} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{array}$$

4 neighbors of l

$$\begin{array}{ccc} & l_1 & \\ l_4 & l & l_2 \\ & l_3 & \end{array}$$

- Free boundary condition

$$\begin{aligned} \partial l &= \{h, p, k\} \\ \partial p &= \{l, o\} \end{aligned}$$

- Toriodal boundary condition (asteroids)

$$\begin{aligned} \partial l &= \{h, i, p, k\} \\ \partial p &= \{l, m, d, o\} \end{aligned}$$

- Reflective boundary condition

$$l_1 = h,\ l_2 = k,\ l_3 = p,\ l_4 = k$$
$$p_1 = l,\ p_2 = o,\ p_3 = l,\ p_4 = o$$

Connected Sets

- Let $c(s)$ be a neighborhood system such that $c(s) \subset \partial s$.
  - For example, $c(s) = \{r \in \partial s : X_r = X_s\}$
  - Computation of $c(s)$ might be very difficult, but we won't worry about that now.
- Definition: A region $R \subset S$ is said to be connected under neighborhood system $c(s)$ if for all $s, r \in R$ their exists a sequency of M pixels, $s_1, \cdots, s_M$ such that

$$s_1 \in c(s), s_2 \in c(s_1), \cdots, s_M \in c(s_{M-1}), r \in c(s_M)$$

  i.e. there is a connected path from s to r.

Example of Connect Sets

- Consider the following image X_s

$$
\begin{array}{cccccc}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{array}
$$

$$
\begin{aligned}
S_1 &= \{s : X_s = 1\} \\
S_0 &= \{s : X_s = 0\}
\end{aligned}
$$

- Define $c(s) = \{r \in \partial s : X_r = X_s\}$
- Result
  - 4-point neighborhood $\Rightarrow$ S_0 and S_1 are not connected sets
  - 8-point neighborhood $\Rightarrow$ S_0 and S_1 **are** connected!

Region Growing

- Idea - grow out region from a seed point s_0
- Assume that $c(s)$ is given

$ClassLabel = 1$

Intialize $Y_r = 0$ for $r \in S$

ConnectedSet($s_0, Y, ClassLabel$) {

  $Y_{s_0} = ClassLabel$

  $B = \{s\}$

  While B is not empty {

    $s \leftarrow$ any element of B

    $B \leftarrow B - \{s\}$

    $Y_s \leftarrow ClassLabel$

    $B \leftarrow B \cup \{r : r \in c(s) \text{ and } Y_r = 0\}$

  }

  return(Y)

}

Region Growing Example (1)

The list of
$(i, j) \in B$
(0,0)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	0	0	0	0	0
	2	0	0	0	0	0
	3	0	0	0	0	0
	4	0	0	0	0	0

Region Growing Example (2)

The list of
$(i, j) \in B$
(1,0)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	0	0	0	0
	2	0	0	0	0	0
	3	0	0	0	0	0
	4	0	0	0	0	0

Region Growing Example (3)

The list of
$(i, j) \in B$
(1,1)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	0	0	0	0
	3	0	0	0	0	0
	4	0	0	0	0	0

Region Growing Example (4)

The list of
$(i, j) \in B$
(2,1)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	0	0	0
	3	0	0	0	0	0
	4	0	0	0	0	0

Region Growing Example (5)

The list of
$(i, j) \in B$
(3,1)
(2,2)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	0	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	0	0	0
	4	0	0	0	0	0

Region Growing Example (6)

The list of $(i, j) \in B$

(4,1)
(3,2)
(2,2)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Region Growing Example (7)

The list of
$(i, j) \in B$
(3,2)
(2,2)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Region Growing Example (8)

The list of
$(i, j) \in B$
(2,2)

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Region Growing Example (9)

The list of $(i, j) \in B$ empty

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Connected Components Extraction

- Iterate through each pixel in the image.
- Extract connected set for each unlabeled pixel.

$ClassLabel = 1$

Intialize $Y_r = 0$ for $r \in S$

For each $s \in S$ {

  if($Y_s \neq 0$) {

    ConnectedSet$(s, Y, ClassLabel)$

    $ClassLabel \leftarrow ClassLabel + 1$

  }

}

Connected Components Extraction Example (1)

$s = (i, j); ClassLabel$

$(0, 0); 1$

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	0

Connected Components Extraction Example (2)

$s = (i, j)$; $ClassLabel$

$(0, 1)$; 2

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	2	2	2	2
	1	1	1	2	2	2
	2	0	1	1	2	2
	3	0	1	1	2	2
	4	0	1	2	2	0

Connected Components Extraction Example (3)

$s = (i, j)$; $ClassLabel$

$(2, 0)$; 3

The image X

		j				
		0	1	2	3	4
i	0	1	0	0	0	0
	1	1	1	0	0	0
	2	0	1	1	0	0
	3	0	1	1	0	0
	4	0	1	0	0	1

The segmentation Y

		j				
		0	1	2	3	4
i	0	1	2	2	2	2
	1	1	1	2	2	2
	2	3	1	1	2	2
	3	3	1	1	2	2
	4	3	1	2	2	0

Connected Components Extraction Example (4)

$s = (i, j); ClassLabel$

$(4, 4); 4$

The image X

i \ j	0	1	2	3	4
0	1	0	0	0	0
1	1	1	0	0	0
2	0	1	1	0	0
3	0	1	1	0	0
4	0	1	0	0	1

The segmentation Y

i \ j	0	1	2	3	4
0	1	2	2	2	2
1	1	1	2	2	2
2	3	1	1	2	2
3	3	1	1	2	2
4	3	1	2	2	4