Sampling

1-D

$$\underset{\text{C-T input}}{g(t)} \longrightarrow \boxed{\begin{matrix}\text{sampler}\\ T\end{matrix}} \longrightarrow \underset{\text{D-T output}}{s(n) = g(nT)}$$

- $f_s = \frac{1}{T}$ = sampling frequency

What is the relationship between

$$S(e^{j\omega}) \text{ and } G(f) \;?$$

$$S(e^{j\omega}) = \frac{1}{T}\, \text{rep}_{2\pi}\left[G(\omega/2\pi T) \right]$$

Intuition

- scale frequencies

$$f = 0 \Rightarrow \omega = 0$$

$$f = \frac{1}{2T} = \frac{f_s}{2} \Rightarrow \omega = \pi$$

- Replicate at period 2π
  - $S(e^{j\omega})$ must have period 2π
  - If signal has energy at $f > \frac{f_s}{2}$, then aliasing occurs

- apply gain factor of $\frac{1}{T}$

Lecture 4

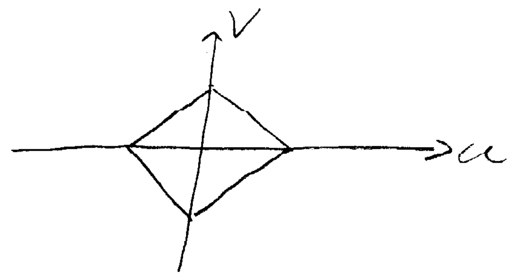
2-D $\quad g(x,y) \rightarrow \boxed{\begin{matrix}\text{sample}\\ T_x \; T_y\end{matrix}} \rightarrow s(m,n)$

$$s(m,n) = g(mT_x, nT_y)$$

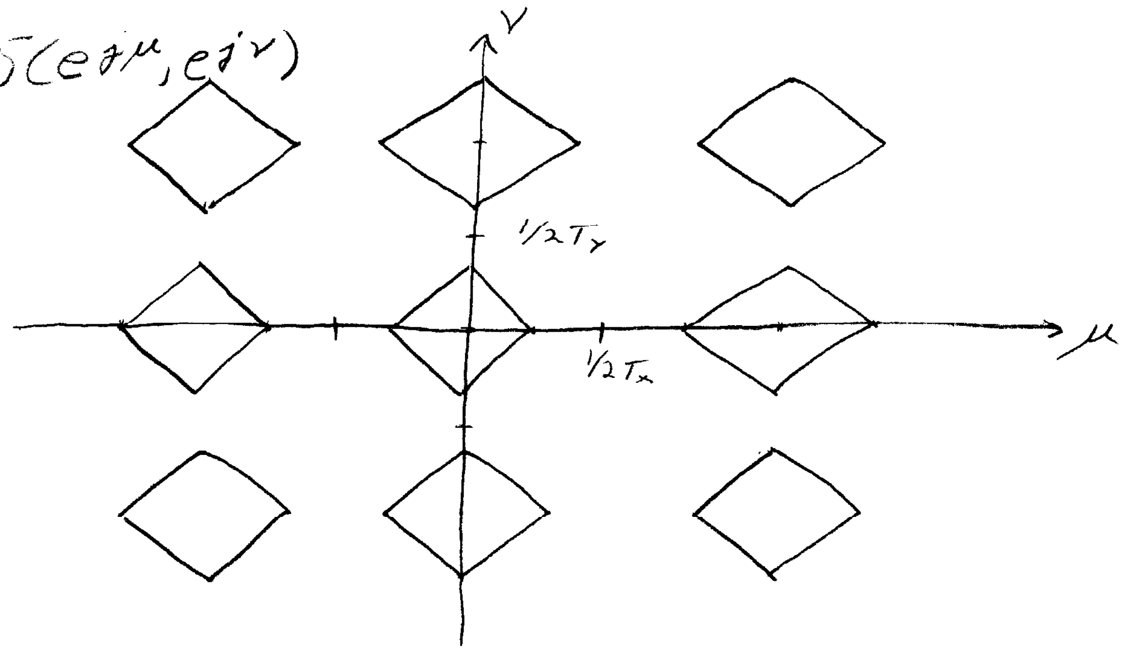
$$S(e^{j\mu}, e^{j\nu}) = \frac{1}{T_x T_y} \, \mathrm{rep}_{2\pi, 2\pi}\left[G\left(\frac{\mu}{2\pi T_x}, \frac{\nu}{2\pi T_y} \right) \right]$$

Example

$G(u,v)$



$S(e^{j\mu}, e^{j\nu})$



Nyquist Condition

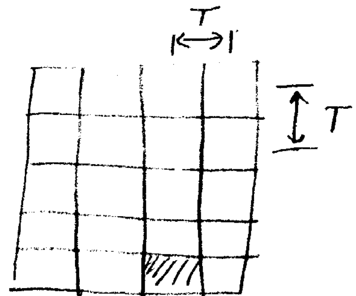
$g(x,y)$ may be uniquely reconstructed from its sampled version $s(n,m)$ if

$$G(u,v) = 0 \quad \text{for all } |u| > \frac{1}{2T_x} \text{ and } |v| > \frac{1}{2T_y}$$

- This condition is sufficient but not necessary.

More on sampling and display

- Typical CCD



each cell collects energy

Let $h(x,y) = \frac{1}{T^2}\,\mathrm{rect}(x/T,\, y/T)$

$= $ profile of cell

- Display model (linear)
  - each pixel controls a single "dot" on the screen
  - Dot profile is $p(x,y)$
  - Dots add linearly

Complete model

$$g(x,y) \rightarrow \boxed{\begin{array}{c}\text{filter}\\ h(x,y)\end{array}} \rightarrow \boxed{\begin{array}{c}\text{sample}\\ T,T\end{array}} \rightarrow \boxed{\begin{array}{c}\text{generate}\\ \text{2-D comb}\\ \text{of impulses}\end{array}} \rightarrow \boxed{\begin{array}{c}\text{filter}\\ \text{with}\\ p(-x,-y)\end{array}} \rightarrow f(x,y)$$

Can be shown that: Lecture 16

- $$\tilde{F}(u,v) = \frac{P^*(u,v)}{T_x T_y} \, rep_{\frac{1}{T_x} \frac{1}{T_y}} \left[H(u,v) \, G(u,v) \right]$$

effect of display dot

aliasing

sampling appature

Assume:

1) Signal band limited to Nyquist frequency
2) $P^*(u,v)$ is band limited to Nyquist frequency

then

- $$\tilde{F}(u,v) = \underbrace{\frac{P^*(u,v)}{T_x T_y} H(u,v)}_{\text{Low pass filter distortion}} \; G(u,v)$$

$$H(u,v) = \mathrm{sinc}(u/f_s, v/f_s)$$

- Result: Scanned images are blurry and need to be sharpened.